APPLICATION OF MESHLESS METHOD FOR BEHAVIOR ANALYSIS OF JOINTED ROCK MASS

M. Hajiazizi[1], N. Hataf[2], F. Daneshmand[3] and A. Ghahramani[4]

[1] *Ph.D student, School of Engineering, Shiraz University, IRAN*
(nhataf@shirazu.ac.ir: hajiazizi@yahoo.com)

[2] *Associate Professor, School of Engineering, Shiraz University, IRAN*
[3] *Assitant Professor, School of Engineering, Shiraz University, IRAN*
[4] *Professor,School of Engineering, Shiraz University, IRAN.*

In analyzing the mechanical behavior of engineering materials, the methods based on meshes such as finite element (FEM) or finite difference (FDM) when large deformations and large number of discontinuities are involved, are not suitable. The conventional process for dealing with moving discontinuities in these methods is to remesh the domain in each step of the process which leads in reduction of speed and degradation of accuracy and complexity of the calculations. To overcome these problems the meshless methods are introduced and applied which are found very suitable for problems with change of geometry. There are a number of meshless methods, such as the Element Free Galerkin (EFG) method. In this paper EFG method is used for the mechanical analysis of intact rock mass and jointed rock mass, the modified penalty method is used for essential boundary condition that is a new application for this method and good results were obtained. Also the time of calculation for EFG method is less than that for the methods based on meshes. It is found that the present method is more suitable for modeling discontinuities of rock mass in two–dimensional and particular three-dimensional domains than conventional methods based on meshes.

Keywords: Element Free Galerkin Method; Jointed Rock Mass; Meshless Method.

1. Introduction

The behavior of jointed rock mass is mostly controlled by the characteristics of rock discontinuities. Crack propagations and large deformations in jointed rock mass are common phenomenon. It is required to model these large behaviors properly with arbitrary paths. Many numerical methods have been proposed for jointed rock mass such as Finite Element Method, Finite Difference Method, Joint Element, Boundary Element Method, Discrete Element Method, Discontinuous Deformation Analysis, Rigid Finite Element Method and several other methods. These methods are mesh-based and require remeshing the domain in each step of the evolution as geometry changes. Consequently mesh generation in each step of the evolution leads to degradation of accuracy and complexity in the computer program and it is also a time consuming task. Thus the mesh-based methods are not efficient for problems involving large deformation and crack propagation that require the continuous remeshing of the domain. Although several strategies have been developed to maintain a reasonable mesh shape, such as the Arbitrary Lagrangian–Eulerian (ALE) method, extra computational effort and difficulties are also introduced (Noh, 1964). Over the past three decades, many researchers have come to realize that so called meshless methods can be developed that eliminate the meshes and their difficulties. Fries and Matthies (Fries & Matthies, 2004) published a special paper on classification and overview of meshfree methods. They described type of meshless methods such as Smooth Particle Hydrodynamics (SPH), Diffuse Element Method (DEM), Element Free Galerkin (EFG), Least Squares Meshfree Method (LSMM), Meshfree Local Petrov Galerkin (MLPG), Local Boundary Integral Equation (LBIE), Partition of Unity Methods (PUM), hp clouds, Natural Element Method (NEM), Meshless Finite Element Method (MFEM), Reproducing Kernel Element Method (RKEM). In this paper the application of EFG method for mechanical analysis of jointed rock mass along with enforcing the essential boundary conditions using is described. It is shown that excellent results are obtained and less calculation time compare to FDM is consumed.

2. Moving Least Squares (MLS)

The moving least squares approximation were developed by Lancaster and Salkauskas(Lancaster & Salkauskas, 1981) for data fitting and surface construction. Nayroles et al. (Nayroles et al., 1992) has used MLS to construct shape function and named it diffuse element method (DEM). DEM was modified by Belytschko and who named it the element free Galerkin method. The MLS approximation $u^h(\mathbf{x})$ is defined in the form of

$$u^h(\mathbf{x})=\sum_{j=1}^{m} p_j(\mathbf{x})a_j(\mathbf{x}), \quad (1)$$

or

$$u^h(\mathbf{x})=\mathbf{p}^T(\mathbf{x})\mathbf{a}(\mathbf{x}), \quad (2)$$

where $p_j(\mathbf{x})$ are monomials of basis function in the space coordinates

$$\mathbf{x}^T=[x,y,z], \quad (3)$$

$\mathbf{p}^T(\mathbf{x})$ in 2D space is provided by

$$\mathbf{p}^T(x,y)=[1,x,y,xy,x^2,y^2, \dots ,x^m,y^m], \quad (4)$$

where m is the number of terms of monomials (polynomial basis) and a(x) is a vector of coefficients and is obtained at any point x by minimizing J.

$$J=\sum_{I=1}^{m} w(\mathbf{x}-\mathbf{x}_I)[\mathbf{p}^T(\mathbf{x}_I)\mathbf{a}(\mathbf{x})-u_I]^2 , \quad (5)$$

where J is a function of weighted residual and constructed using the approximated values of the field function and $\mathbf{u}_I=\mathbf{u}(\mathbf{x}_I)$.

The stationary of J with a(x)

$$\mathbf{A}(\mathbf{x})\mathbf{a}(\mathbf{x})=\mathbf{B}(\mathbf{x})\mathbf{u} , \quad (6)$$

or

$$\mathbf{a}(\mathbf{x})=\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u} , \quad (7)$$

where

$$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w_I(\mathbf{x})\mathbf{p}^T(\mathbf{x}_I)\mathbf{p}(\mathbf{x}_I) , \quad (8)$$

$$\mathbf{B}(\mathbf{x})=[w_1(\mathbf{x})\mathbf{p}(\mathbf{x}_1) , w_2(\mathbf{x})\mathbf{p}(\mathbf{x}_2) , \dots , w_n(\mathbf{x})\mathbf{p}(\mathbf{x}_n)] , \quad (9)$$

$$\mathbf{u}= [u_1, u_2 , \dots , u_n] . \quad (10)$$

Substituting the equation (7) into (1) leads to

$$u^h(\mathbf{x})=\sum_{I=1}^{n}\sum_{j=1}^{m} p_j(\mathbf{x})(\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}))_{jI}u_I , \quad (11)$$

or

$$u^h(\mathbf{x})=\sum_{I=1}^{n} \Phi_I(\mathbf{x})u_I , \quad (12)$$

where the MLS shape function $\Phi_I(\mathbf{x})$ is defined by

$$\Phi_I(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})(\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}))_{jI} , \quad (13)$$

or

$$\mathbf{\Phi}(\mathbf{x})=\mathbf{p}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}) . \quad (14)$$

The partial derivative of ΦI(x) can be obtained as follows

$$\Phi_{I,i}=\sum_{j=1}^{m}\left\{ p_{j,i}(\mathbf{A}^{-1}\mathbf{B})_{jI}+p_j\mathbf{A}^{-1}(\mathbf{B}_{,i}-\mathbf{A}_{,i}\mathbf{A}^{-1}\mathbf{B})_{jI} \right\} . \quad (15)$$

The weight function $w_I(\mathbf{x})$ is positive and an important coefficient. In this paper it is considered as (Belytschko et al., 1994)

$$w_I(\mathbf{x})= \begin{cases} 1-6s^2+8s^3-3s^4 & \text{for} \quad d_I<=r \\ 0 & \text{for} \quad d_I>r \end{cases} , \quad (16)$$

where $s=d_I/r$, $d_I=||\mathbf{x}-\mathbf{x}_I||$, r =influence domain.

The weight function is large for $\mathbf{x}_I$ close to $\mathbf{x}$ and is small otherwise and is zero for out of influence domain.

3. Equilibrium Equation

The equilibrium equation can be obtained from variational principles. The functional of the total potential energy of a jointed rock mass is given by (Zhang , 2000)

$$\Pi=\Pi_b+\Pi_j+\Pi_f+\Pi_p \ , \tag{17}$$

where Π_b is the elastic strain energy of block, Π_j is the elastic strain energy of joints. Π_f is the potential energy of the body force f_e, Π_p is the potential energy of the concentrated force p. They are given by

$$\Pi_b=\sum_e t_e\iint_{\Omega e}0.5\varepsilon^T\mathbf{D}_b\,\varepsilon\,dxdy=0.5\sum_e\mathbf{u}_b^T(t_e\iint_{\Omega e}\mathbf{B}_b^T\,\mathbf{D}_b\mathbf{B}_b\,dxdy)\mathbf{u}_b \ , \tag{18}$$

$$\Pi_j=\sum_k t_k\int_{\beta k}0.5\delta^T\mathbf{D}_j\,\delta ds=0.5\sum_k\mathbf{u}_j^T(t_k\int_{\beta k}\mathbf{B}_j^T\,\mathbf{D}_j\mathbf{B}_j ds)\mathbf{u}_j \ , \tag{19}$$

$$\Pi_f=-\sum_e\iint_{\Omega e}\mathbf{u}^T\mathbf{f}_e\,dxdy=-\sum_e\mathbf{u}_b^T(\iint_{\Omega e}\mathbf{N}^T\,\mathbf{f}_e\,dxdy) \ , \tag{20}$$

$$\Pi_p=-\sum_m\mathbf{u}^T\mathbf{p}_m=-\sum_m\mathbf{u}_b^T(\mathbf{N}^T\,\mathbf{p}_m) \ . \tag{21}$$

We can obtain equilibrium equation from the stationary of functional Π in (17). The equilibrium equation of jointed rock mass is given by

$$\mathbf{KU=P} \ , \tag{22}$$

where

$$\mathbf{K}=\sum_e t_e\iint_{\Omega e}\mathbf{B}_b^T\,\mathbf{D}_b\mathbf{B}_b\,dxdy+\sum_k t_k\int_{\beta k}\mathbf{B}_j^T\,\mathbf{D}_j\mathbf{B}_j ds \ , \tag{23}$$

$$\mathbf{P}=\sum_e\iint_{\Omega e}\mathbf{N}^T\,\mathbf{f}_e\,dxdy+\sum_m\mathbf{N}^T\mathbf{p}_m \ , \tag{24}$$

$$\mathbf{U}=[u_1,v_1,u_2,v_2,\ldots,u_n,v_n]^T \ , \tag{25}$$

where t_e, t_k are the thickness of the block e and the joint k, respectively, n is the total number of nodes in the problem domain, $\mathbf{U}$ is displacement vector related to total number of nodes in the problem domain.

4. Displacement, Strain and Stress in Blocks

The displacement of any point into any blocks is obtained by

$$\mathbf{u}=\mathbf{N}.\mathbf{u}_b \ , \tag{26}$$

where

$$\mathbf{N}=\begin{pmatrix}\Phi_1(\mathbf{x})\ ,0\ ,\Phi_2(\mathbf{x})\ ,0\ ,\ldots,\Phi_n(\mathbf{x})\ ,0\\ \\ 0\ ,\Phi_1(\mathbf{x})\ ,0\ ,\Phi_2(\mathbf{x})\ ,0\ ,\ldots,\Phi_n(\mathbf{x})\end{pmatrix} \ , \tag{27}$$

and $\mathbf{u}_b$ is the displacement vector of influence domain $\mathbf{x}$. The strain and stress at any point $\mathbf{x}$ are given by

$$\boldsymbol{\varepsilon}=\mathbf{B}_b\mathbf{u}_b \ , \tag{28}$$

$$\boldsymbol{\sigma}=\mathbf{D}_b\mathbf{B}_b\mathbf{u}_b \ , \tag{29}$$

where

$$\mathbf{B}_b=\begin{pmatrix}\Phi_{1,x}(\mathbf{x}) & ,0, & \Phi_{2,x}(\mathbf{x}), & 0,\ldots, & \Phi_{n,x}(\mathbf{x}), & 0\\ 0, & \Phi_{1,y}(\mathbf{x}) & ,0, & \Phi_{2,y}(\mathbf{x}) & ,0,\ldots, & \Phi_{n,y}(\mathbf{x})\\ \Phi_{1,y}(\mathbf{x}), & \Phi_{1,x}(\mathbf{x}) & ,\ldots, & & \Phi_{n,y}(\mathbf{x}), & \Phi_{n,x}(\mathbf{x})\end{pmatrix} \ , \tag{30}$$

and

$$\mathbf{D}_b=(E/(1-\upsilon^2))\begin{pmatrix}1 & \upsilon & 0\\ \upsilon & 1 & 0\\ 0 & 0 & (1-\upsilon)/2\end{pmatrix} \quad \text{for plain stress state,} \tag{31}$$

$$\mathbf{D}_b = (E/(1-2\upsilon)(1+\upsilon)) \begin{pmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & (1-2\upsilon)/2 \end{pmatrix} \text{ for plain strain state,} \quad (32)$$

and

$$\boldsymbol{\varepsilon}=[\varepsilon_x,\varepsilon_y,\varepsilon_{xy}]^T , \quad (33)$$

$$\boldsymbol{\sigma}=[\sigma_x, \sigma_y, \sigma_{xy}]^T . \quad (34)$$

5. Displacement, Strain and Stress in Joints

The Contribution of joints in the equilibrium equation **KU=P** is

$$\mathbf{K_j}=\sum_k t_k \int_{\beta k} \mathbf{B_j}^T \mathbf{D_j B_j} ds . \quad (35)$$

We can write

$$\mathbf{K}=\mathbf{K_b} +\mathbf{K_j} , \quad (36)$$

where $\mathbf{K_b}$ and $\mathbf{K_j}$ are stiffness matrix for block and joint respectively. For the mechanical analysis of intact rock mass $\mathbf{K_j}$ is eliminated**.** For the mechanical analysis of jointed rock mass it is required to calculate $\mathbf{K_j}$**.** In equation (35) $\mathbf{B_j}$ and $\mathbf{D_j}$ are moment matrix and elasticity matrix of the joint respectively. The moment matrix of the joint can be defined as:

$$\mathbf{B_j}=[\,\underline{0}\,, \dots , -\mathbf{LN_i} , \underline{0}\, , \dots , \mathbf{LN_j} , \underline{0} , \dots] , \quad (37)$$

where **L** is the transform matrix and $\mathbf{N_i}$**,** $\mathbf{N_j}$ are the shape functions matrices of the i and j surfaces of joint (Fig. 1). Fig. 2 shows off and on nodes due to change of geometry.

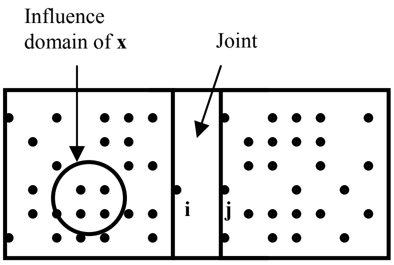


Fig. 1. Two block connected with joint and nodes of meshless

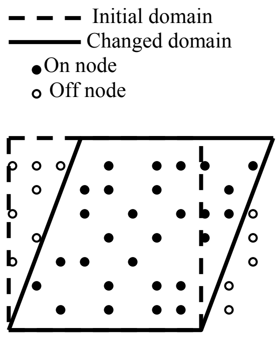


Fig. 2. Changed domain with initial domain

The elasticity matrix $\mathbf{D_j}$ of joint can be defined as

$$\mathbf{D_j}=\begin{pmatrix} k_s & 0 \\ 0 & k_n \end{pmatrix}. \quad (38)$$

The relative displacement vector δ of the joint is given as follows

$$\delta= [\delta_n , \ \delta_s]=\tilde{u}_j - \tilde{u}_i=\mathbf{B_j}\, \mathbf{u_j} , \quad (39)$$

where $\tilde{u}_i$ and $\tilde{u}_j$ are the displacement vectors of joints i and j from the surface of joint, respectively. Also $\mathbf{u_j}$ is

$$\mathbf{u_j}=[u_i^T , \ u_j^T]^T , \quad (40)$$

where u_i and u_j are displacements of point i and point j on the joint surfaces. The strain vector of joint can be defined as

$$\begin{pmatrix} \varepsilon_n \\ \gamma_{ns} \end{pmatrix} = \delta/h=(1/h)\mathbf{B_j u_j} , \quad (41)$$

where ε_n is the normal strain and γ_{ns} is the shear strain. The stress vector of joint is

$$\begin{pmatrix} \sigma_n \\ \tau_s \end{pmatrix} = \mathbf{D_j}\,\delta\,, \tag{42}$$

where σ_n is the normal stress and τ_s is the shear stress.

6. The equilibrium equation and enforcing the essential boundary condition

The nodal value of the interpolation function $u^h(\mathbf{x})$ in element free Galerkin method is not equal to the nodal value of the function $u(\mathbf{x})$.

This is because that the shape function of EFG method is not equal to kronecker delta. Namely:

$$\Phi_I(\mathbf{x}_J) \neq \delta_{IJ}\,. \tag{43}$$

The essential boundary condition thus should be imposed. A simple and efficient way for imposing essential boundary condition is penalty method. The essential boundary condition is:

$$\mathbf{u}=\hat{\mathbf{u}} \qquad \text{on} \qquad \mathbf{\Gamma_u}\,, \tag{44}$$

where $\hat{\mathbf{u}}$ is the prescribed displacement on boundary $\mathbf{\Gamma_u}$.

The equation is obtained from weak form Galerkin with enforcing the essential boundary condition and using penalty method as (Liu, 2002):

$$\int_\Omega \delta(\mathbf{Lu})^T c(\mathbf{Lu})d\Omega - \int_\Omega \delta\mathbf{u}^T .\, b\, d\Omega - \int_{\Gamma t}\delta\mathbf{u}^T .\, \tilde{t}\, d\Gamma - \delta\int_{\Gamma u}(.5)(u-\bar{u})^T.\alpha.(u-\bar{u})d\Gamma=0\,. \tag{45}$$

Applying mathematical calculation on equation (45), the final equilibrium equation is

$$[\mathbf{K}+\mathbf{K}_\alpha]\,\mathbf{U}=\mathbf{P}+\mathbf{P}_\alpha\,, \tag{46}$$

where $\mathbf{K}_\alpha$ and $\mathbf{P}_\alpha$ are obtained for the essential boundary condition using penalty method, as:

$$\mathbf{K}_\alpha=\alpha\int_{\Gamma u}\mathbf{N}^T\mathbf{N}\, d\Gamma\,, \tag{47}$$

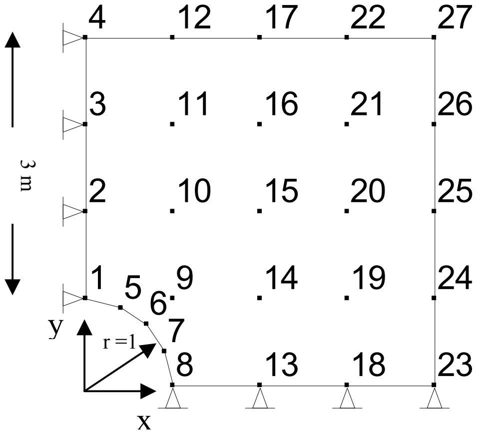
$$\mathbf{P}_\alpha=\alpha\int_{\Gamma u}\mathbf{N}^T\bar{\mathbf{u}}\, d\Gamma\,, \tag{48}$$

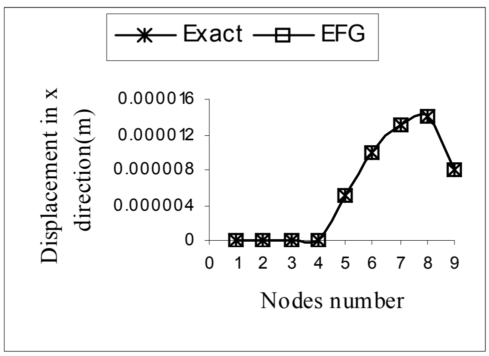
where α is penalty factor.

7. Numerical Examples

Example 1: Infinit plate with a circular hole

The first example a plate with a circular hole to a uniform tension, σ=100N/m, in the x direction is considered as shown in Fig. 3. In the computation a plane strain state with E=20 MPa, ν=0.3 and penalty parameter $\alpha=10^5$ E is considered. 27 nodes is considered in the domain as shown in Fig. 3. The weight function is Quarticspline. To compare exact solution with EFG method for all nodes in Figs. 4 to 6 are prepared.





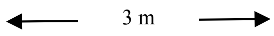


Fig. 3. The plate subjected to a tensile in the horizontal direction

Fig. 4. Displacement in x direction for 1 to 9 nodes

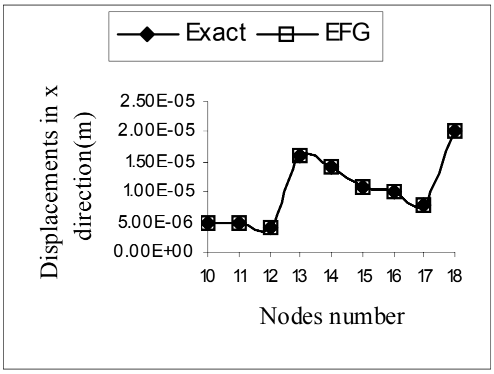


Fig. 5. Displacement in x direction for 10 to 18 nodes

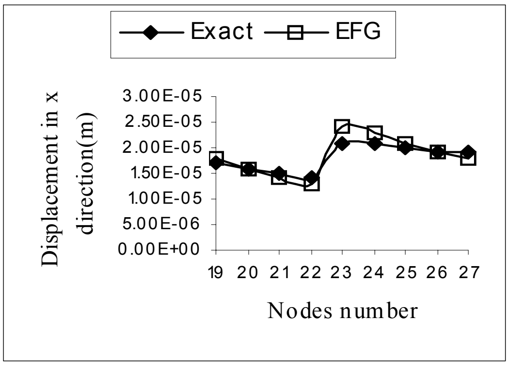
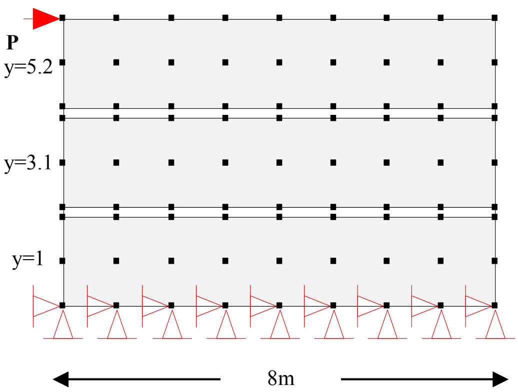


Fig. 6. Displacement in x direction for 19 to 27 nodes

As it can be seen from these figures good agreements exist between two set of results.

Example 2: Jointed rock mass subjected to horizontal load P

A jointed rock mass subjected to horizontal force (P=100 KN) is shown Fig. 7. The length of rock mass was equal to 8m and its high was equal to 6.2 m. The joint properties are K_n=K_s=.5e8 Pa/m. The rock properties are ν=.3, E=2e7 Pa, cohesion=15 KPa, friction=0˚ and dilation angle=0˚. The number of mesh for any block is 2*8 and total number of nodes is 81. This problem is solved in plane strain state. The number of Quadrature points in each mesh=10*10, penalty factor = 1e5 and weight function was equal to Quarticspline, influence radius of any node=1.42. The distances between nodes in horizontal and vertical direction are 1m. The vertical distance between nodes of joint is 0.1m. This problem is compared with FDM. The results are shown in Figs. 8 to 10.The time of calculation of EFG is much less than FDM, and the results are comparable.



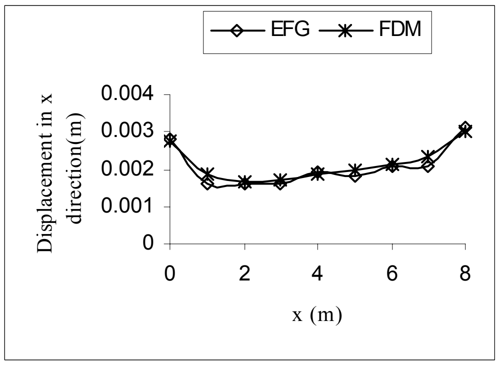
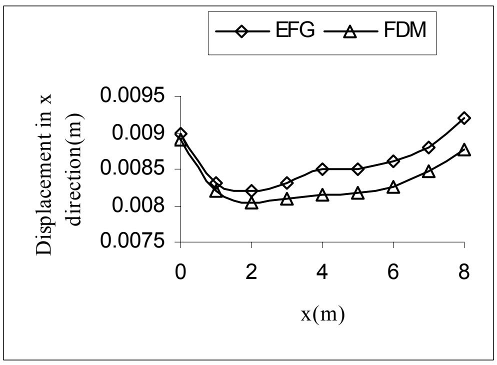


Fig. 7. A jointed rock mass subjected to horizontal load

Fig. 8. Displacement of nodes horizon in y=1



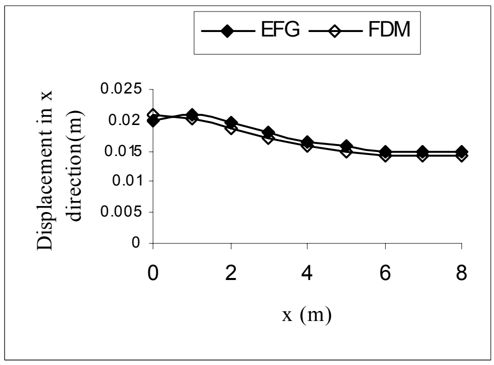


Fig. 9. Displacement of nodes horizon in y=3.1 Fig. 10. Displacement of nodes horizon in y=5.2

8. Conclusions

The mesh-based methods for large deformations and large number of discontinuities problems leads in reduction of speed of calculation, degradation of accuracy and complexity of the calculations. To overcome these problems the meshless methods are introduced that no finite element mesh is required and only a number of points are distributed and applied which are found very suitable for problems with change of geometry such as jointed rock mass problems. The EFG method is an effective method for nonlinear problems. Using the element free Galerkin method for jointed rock mass in the elastic state with modified penalty method has shown good results. Comparing of exact solution and EFG method in examples particularly in example 1 has shown efficiency and validity of EFG method results.

9. References

Belytschko, T. and Lu, YY. and Gu, L., (1994). "Element-Free Galerkin Method", *International Journal for Numerical Methods in Engineering*, 37, 229 – 256.

Belytschko, T. and Organ, D. and Gerlach C. (2000). "Element –free Galerkin methods for dynamics fracture in concrete". *Comput. Method Appl. Mech. Engrg*., 187, 385-399.

Fries, T. P. and Matthies, H.-G. (2004). "Classification and Overview of Meshfree Methods". Braunschweig Institute fűr Wissenschaftliches Rechnen Technische Universität Braunschweig.

Lancaster, P. and salkauskas, K. (1981). "Surface generated by moving least squares methods". *Math. Comp.*, 37, 141-158.

Liu, G. R. (2002). ″Mesh Free Methods: Moving beyond the Finite Element Method″, Florida: CRC Press.

Nayroles, B. and Touzot, G. and Villon, P. (1992). "Generalizing the finite element method: diffuse approximation and diffuse elements". *Comput. Mech.,* 10, 307-318.

Noh, W. (1964). "A Time –Dependent Two-Space-Dimensional Coupled Eulerian Code". *In Methods in Computational physics,* 3: Academic Press.

Zhang, X. and Lu, M. and Wegner, J.L. (2000). "A 2-D meshless model for jointed rock structures." *Int. J. for numerical methods in engrg.*, Vol. 47, pp. 1649-1661.