ELASTIC-PLASTIC ANALYSIS OF JOINTED ROCK MASS USING MESHLESS METHOD

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Analysis of rock masses with plenty of weakness planes and joints with numerical methods based on meshes encounters difficulties. In the analysis when even only a few meshes are involved, mesh generation can consume more time and effort compared to the construction and solution of the discrete set of equations. Continuous remeshing of the domain in order to avoid the break down of the calculation due to excessive mesh distortion is other concern. These problems lead in reducing accuracy and increasing complexity in computer codes. To overcome these shortcomings recently the meshfree methods have been developed and found applications in different engineering fields. In this paper the application of mesh-free (Element Free Galerkin) method in analyzing the nonlinear and elastoplastic behavior of jointed rock mass is presented. This is a new application of such method. In this method, there is no need to use meshes or elements for field variable interpolation. The nodes remain constant while the geometry of the domain is changing, therefore saving the time and computational effort in analyzing process. A few examples of application of the method in predicting the plastic behavior of jointed rocks are presented, good results and the suitability of the method is highlighted.

Keywords: Nonlinear analysis; Elastic-plastic analysis; Nonlinear; Jointed rock mass; Meshless; Element free Galerkin.

Introduction

The nonlinear elastic-plastic analysis of jointed rock mass is true behavior analysis of rock. Large deformations in jointed rock mass are common phenomenon. Many numerical methods have been proposed for jointed rock mass such as Joint Element, Distinct Element Method, Discontinuous Deformation Analysis, Rigid Finite Element Method and several other methods. These methods are mesh-based and require remeshing the domain in each step of the evolution as geometry changes. Consequently, mesh generation in each step of the evolution leads to degradation of accuracy and complexity in the computer program and it is also a time consuming task. Thus the mesh-based methods are not efficient for problems involving large deformation and crack propagation that require the continuous remeshing of the domain. Over the past three decades, many researchers have come to realize that so called meshless methods can be developed that eliminate the meshes and their difficulties. Fries and Matthies (Fries & Matthies, 2004) published a special paper on classification and overview of meshfree methods. They described type of meshless methods such as Smooth Particle Hydrodynamics (SPH), Diffuse Element Method (DEM), Element Free Galerkin (EFG), Least Squares Meshfree Method (LSMM), Meshfree Local Petrov Galerkin (MLPG), Local Boundary Integral Equation (LBIE), Partition of Unity Methods (PUM), hp clouds, Natural Element Method (NEM), Meshless Finite Element Method (MFEM), Reproducing Kernel Element Method (RKEM). In this paper the application of EFG method for nonlinear elastic-plastic analysis of jointed rock mass is described, the elastic portion it similar to the paper of "*Application of Element Free Galerkin Method for Two Dimensional Analysis of Jointed Rock Mass* " that we submitted for ARMS-4 and plastic portion is added in this paper. It is shown that excellent results are obtained and less calculation time is consumed.

2. Moving Least Squares (MLS)

The MLS approximation $u^h(\mathbf{x})$ is defined in the form of:

$$u^h(\mathbf{x})=\sum_{j=1}^{m} p_j(\mathbf{x})a_j(\mathbf{x}), \tag{1}$$

where $p_j(\mathbf{x})$ are monomials of basis function in the space coordinates

$$\mathbf{x}^T=[x,y,z]. \tag{2}$$

$\mathbf{p}^T(\mathbf{x})$ in 2D space is provided by

$$\mathbf{p}^T(x,y)=[1,x,y,xy,x^2,y^2, \dots ,x^m,y^m], \tag{3}$$

where m is the number of terms of monomials (polynomial basis) and $\mathbf{a}(\mathbf{x})$ is a vector of coefficients and is obtained at any point x by minimizing J.

$$J=\sum_{I=1}^{m} w(\mathbf{x}-\mathbf{x}_I)[\mathbf{p}^T(\mathbf{x}_I)\mathbf{a}(\mathbf{x})-u_I]^2 , \tag{4}$$

where J is a function of weighted residual and constructed using the approximated values of the field function and $\mathbf{u_I}=\mathbf{u}(\mathbf{x}_I)$. The stationary of J with $\mathbf{a}(\mathbf{x})$

$$\mathbf{a}(\mathbf{x})=\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u}, \tag{5}$$

where

$$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w_I(\mathbf{x})\mathbf{p}^T(\mathbf{x}_I)\mathbf{p}(\mathbf{x}_I), \tag{6}$$

$$\mathbf{B}(\mathbf{x})=[w_1(\mathbf{x})\mathbf{p}(\mathbf{x}_1) , w_2(\mathbf{x})\mathbf{p}(\mathbf{x}_2) , \dots , w_n(\mathbf{x})\mathbf{p}(\mathbf{x}_n)], \tag{7}$$

$$\mathbf{u}= [u_1, u_2 , \dots , u_n]. \tag{8}$$

Substituting the equation (5) into (1) leads to

$$u^h(\mathbf{x})=\sum_{I=1}^{n} \Phi_I(\mathbf{x})u_I , \tag{9}$$

where the MLS shape function $\Phi_I(\mathbf{x})$ is defined by

$$\Phi_I(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})(\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}))_{jI} . \tag{10}$$

The partial derivative of $\Phi_I(x)$ can be obtained as follows

$$\Phi_{I,i}=\sum_{j=1}^{m}\left\{ p_{j,i}(\mathbf{A}^{-1}\mathbf{B})_{jI}+p_j\mathbf{A}^{-1}(\mathbf{B}_{,i}-\mathbf{A}_{,i}\mathbf{A}^{-1}\mathbf{B})_{jI} \right\}. \tag{11}$$

The weight function $w_I(\mathbf{x})$ is positive and an important coefficient. In this paper it is considered as (Belytschko et al., 1994)

$$w_I(\mathbf{x})=\begin{cases} 1-6s^2+8s^3-3s^4 & \text{for} \quad d_I<=r \\ \\ 0 & \text{for} \quad d_I>r \end{cases} , \tag{12}$$

where $s=d_I/r$, $d_I =||\mathbf{x}-\mathbf{x}_I||$, r =influence domain.

The weight function is large for $\mathbf{x}_I$ close to $\mathbf{x}$ and is small otherwise and is zero for out of influence domain.

3. Equilibrium Equation

The equilibrium equation can be obtained from variational principles. The functional of the total potential energy of a jointed rock mass is given by (Zhang, 2000)

$$\Pi=\Pi_b+\Pi_j+\Pi_f+\Pi_p\ , \tag{13}$$

where Π_b is the elastic strain energy of block, Π_j is the elastic strain energy of joints. Π_f is the potential energy of the body force f_e, Π_p is the potential energy of the concentrated force p. They are given by

$$\Pi_b=\sum_e t_e\iint_{\Omega e}0.5\varepsilon^T\mathbf{D}_b\ \varepsilon\ dxdy=0.5\sum_e\mathbf{u}_b^T(t_e\iint_{\Omega e}\mathbf{B}_b^T\ \mathbf{D}_b\mathbf{B}_b\ dxdy)\mathbf{u}_b\ , \tag{14}$$

$$\Pi_j=\sum_k t_k\int_{\beta k}0.5\delta^T\mathbf{D}_j\ \delta ds=0.5\sum_k\mathbf{u}_j^T(t_k\int_{\beta k}\mathbf{B}_j^T\ \mathbf{D}_j\mathbf{B}_j ds)\mathbf{u}_j\ , \tag{15}$$

$$\Pi_f=-\sum_e\iint_{\Omega e}\mathbf{u}^T\mathbf{f}_e\ dxdy=-\sum_e\mathbf{u}_b^T(\iint_{\Omega e}\mathbf{N}^T\ \mathbf{f}_e\ dxdy)\ , \tag{16}$$

$$\Pi_p=-\sum_m\mathbf{u}^T\mathbf{p}_m=-\sum_m\mathbf{u}_b^T(\mathbf{N}^T\ \mathbf{p}_m). \tag{17}$$

We can obtain equilibrium equation from the stationary of functional Π in (13). The equilibrium equation of jointed rock mass is given by:

$$\mathbf{KU}=\mathbf{P}\ , \tag{18}$$

where

$$\mathbf{K}=\sum_e t_e\iint_{\Omega e}\mathbf{B}_b^T\ \mathbf{D}_b\mathbf{B}_b d_x d_y+\sum_k t_k\int_{\beta k}\mathbf{B}_j^T\ \mathbf{D}_j\mathbf{B}_j ds\ , \tag{19}$$

$$\mathbf{P}=\sum_e\iint_{\Omega e}\mathbf{N}^T\ \mathbf{f}_e\ dxdy+\sum_m\mathbf{N}^T\mathbf{p}_m\ , \tag{20}$$

$$\mathbf{U}=[u_1,v_1,u_2,v_2,\ldots,u_n,v_n]^T\ , \tag{21}$$

where t_e, t_k are the thickness of the block e and the joint k, respectively, n is the total number of nodes in the problem domain, **U** is displacement vector related to total number of nodes in the problem domain.

4. Displacement, Strain and Stress in Blocks

The displacement of any point into any blocks is obtained by:

$$\mathbf{u}=\mathbf{N}.\mathbf{u}_b\ , \tag{22}$$

where

$$\mathbf{N}=\begin{pmatrix}\Phi_1(\mathbf{x})\ ,0\ ,\Phi_2(\mathbf{x})\ ,0\ ,\ldots,\Phi_n(\mathbf{x})\ ,0\\ 0\ ,\Phi_1(\mathbf{x})\ ,0\ ,\Phi_2(\mathbf{x})\ ,0\ ,\ldots,\Phi_n(\mathbf{x})\end{pmatrix}\ , \tag{23}$$

and $\mathbf{u}_b$ is the displacement vector of influence domain **x.** The strain and stress at any point **x** are given by

$$\boldsymbol{\varepsilon}=\mathbf{B}_b\mathbf{u}_b\ , \tag{24}$$

$$\boldsymbol{\sigma}=\mathbf{D}_b\mathbf{B}_b\mathbf{u}_b\ , \tag{25}$$

where $\mathbf{D}_b$ is elasticity matrix and

$$\mathbf{B}_b=\begin{pmatrix}\Phi_{1,x}(\mathbf{x})\ ,0\ ,\ \Phi_{2,x}(\mathbf{x})\ ,\ 0\ ,\ldots,\ \Phi_{n,x}(\mathbf{x})\ ,\ 0\\ 0\ ,\ \Phi_{1,y}(\mathbf{x})\ ,\ 0\ ,\ \Phi_{2,y}(\mathbf{x})\ ,0\ ,\ldots,\ \Phi_{n,y}(\mathbf{x})\\ \Phi_{1,y}(\mathbf{x})\ ,\Phi_{1,x}(\mathbf{x})\ ,\ \ldots,\ \Phi_{n,y}(\mathbf{x})\ ,\ \Phi_{n,x}(\mathbf{x})\end{pmatrix}\ , \tag{26}$$

and

$$\boldsymbol{\varepsilon}=[\varepsilon_x,\varepsilon_y,\varepsilon_{xy}]^T , \tag{27}$$

$$\boldsymbol{\sigma}=[\sigma_x, \sigma_y, \sigma_{xy}]^T , \tag{28}$$

5. Displacement, Strain and Stress in Joints

The Contribution of joints in the equilibrium equation **KU=P** is

$$\mathbf{K_j}=\sum_k t_k \int_{\beta k} \mathbf{B_j}^T \mathbf{D_j}\mathbf{B_j}ds , \tag{29}$$

we can write

$$\mathbf{K}=\mathbf{K_b} +\mathbf{K_j} , \tag{30}$$

where $\mathbf{K_b}$ and $\mathbf{K_j}$ are stiffness matrix for block and joint respectively. For the mechanical analysis of intact rock mass $\mathbf{K_j}$ is eliminated**.** For the mechanical analysis of jointed rock mass it is required to calculate $\mathbf{K_j}$**.** In equation (29) $\mathbf{B_j}$ and $\mathbf{D_j}$ are moment matrix and elasticity matrix of the joint respectively. The moment matrix of the joint can be defined as:

$$\mathbf{B_j}=[\underline{0} , \ldots , -\mathbf{LN_i} , \underline{0} , \ldots , \mathbf{LN_j} , \underline{0} , \ldots] , \tag{31}$$

where $\mathbf{L}$ is the transform matrix and $\mathbf{N_i}$**,** $\mathbf{N_j}$ are the shape functions matrices of the i and j surfaces of joint The elasticity matrix $\mathbf{D_j}$ of joint can be defined as:

$$\mathbf{D_j}= \begin{pmatrix} k_s & 0 \\ 0 & k_n \end{pmatrix} . \tag{32}$$

The strain vector of joint can be defined as

$$\begin{bmatrix} \varepsilon_n \\ \gamma_{ns} \end{bmatrix} = \delta/h=(1/h)\mathbf{B_j}\mathbf{u_j} , \tag{33}$$

where ε_n is the normal strain and γ_{ns} is the shear strain. The stress vector of joint is

$$\begin{pmatrix} \sigma_n \\ \tau_s \end{pmatrix} = \mathbf{D_j}\,\delta , \tag{34}$$

where σ_n is the normal stress and τ_s is the shear stress.

6. The equilibrium equation with enforcing the essential boundary condition

The nodal value of the interpolation function $u^h(\mathbf{x})$ in element free Galerkin method is not equal to the nodal value of the function $u(\mathbf{x})$**.**

This is because that the shape function of EFG method is not equal to kronecker delta. Namely:

$$\Phi_I(\mathbf{x}_J) \neq \delta_{IJ} . \tag{35}$$

The essential boundary condition thus should be imposed. A simple and efficient way for imposing essential boundary condition is penalty method. The essential boundary condition is

$$\mathbf{u}=\hat{\mathbf{u}} \quad \text{on} \quad \boldsymbol{\Gamma_u} , \tag{36}$$

where $\hat{\mathbf{u}}$ is the prescribed displacement on boundary $\boldsymbol{\Gamma_u}$ **.**

The equation is obtained from weak form Galerkin with enforcing the essential boundary condition and using penalty method as (Liu, 2002)

$$\int_\Omega \delta(\mathbf{Lu})^T c(\mathbf{Lu})d\Omega - \int_\Omega \delta\mathbf{u}^T .\ b\ d\Omega - \int_{\Gamma t}\delta\mathbf{u}^T .\ \tilde{t}\ d\Gamma - \delta\int_{\Gamma u}(.5)(u-\bar{u})^T.\alpha.(u-\bar{u})d\Gamma=0 . \quad (37)$$

Applying mathematical calculation on equation (37), the final equilibrium equation is:

$$[\mathbf{K}+\mathbf{K}_\alpha]\ \mathbf{U}=\mathbf{P}+\mathbf{P}_\alpha , \quad (38)$$

where $\mathbf{K}_\alpha$ and $\mathbf{P}_\alpha$ are obtained for the essential boundary condition using penalty method, as

$$\mathbf{K}_\alpha=\alpha\int_{\Gamma u}\mathbf{N}^T\mathbf{N}\ d\Gamma, \quad (39)$$

$$\mathbf{P}_\alpha=\alpha\int_{\Gamma u}\mathbf{N}^T\bar{\mathbf{u}}\ d\Gamma , \quad (40)$$

where α is penalty factor. Ideally it is better to use infinite penalty factor. But if it is taken as infinite or too large, the numerical problems will be encountered. Thus the penalty factor is a number that the constraints be properly enforced. Usually it is equal to 10^5E to 10^8E. We where penalty method is applied good results are obtained as described later in this paper.

7. The model of Elastic-Plastic in Joints

The behavior of joint is elastic if $f(\sigma_n^1, \tau_s^1) < 0$ and $f(\sigma_n^1, \tau_s^1)$ is Mohre-Coulomb failure criteria as shown

$$f=|\tau_s| + \sigma_n \tan\phi - C , \quad (41)$$

where τ_s, σ_n are shear and normal stresses, respectively and Φ, C are friction and cohesion, respectively. The shear displacement increment Δu_s and the normal displacement increment Δu_n include to two part of elastic and plastic as shown follow

$$\Delta u_s = \Delta u_s^e + \Delta u_s^p , \quad (42)$$

$$\Delta u_n = \Delta u_n^e + \Delta u_n^p . \quad (43)$$

The first is assumed the plastic deformation is zero, then the shear, normal displacement increment are

$$\Delta \tau_s = k_s \Delta u_s , \quad (44)$$

$$\Delta \sigma_n = k_n \Delta u_n . \quad (45)$$

Thus new shear and normal stresses are

$$\tau_s^1 = \tau_s + \Delta\tau_s , \quad (46)$$

$$\sigma_n^1 = \sigma_n + \Delta\sigma_n . \quad (47)$$

The behavior of joint is plastic if $f(\sigma_n^1, \tau_s^1) \geq 0$ that necessary to calculate both elastic and plastic deformations. To calculate plastic deformation is used flow rule as shown follow

$$g = |\tau_s| + \sigma_n \tan\psi , \quad (48)$$

where Ψ is the dilation angle. The stress rate relation derived from the normality condition corresponding to a yield function can be written in the general form (Chen, 1975)

$$\varepsilon_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (49)$$

where

$$\lambda = \frac{f(\sigma_n^I, \tau_s^I)}{k_s + k_n \tan\phi \tan\psi} \tag{50}$$

Thus plastic deformation increment is calculated as follows

$$\Delta u_s^p = \lambda \frac{\partial g}{\partial \tau_s} = \lambda \,\mathrm{sgn}(\tau_s) \tag{51}$$

$$\Delta u_n^p = \lambda \frac{\partial g}{\partial \sigma_n} = \lambda \tan(\psi) \tag{52}$$

Therefore the modified stresses are calculated from

$$\tau_s = \tau_s^I - k_s \Delta u_s^p \ , \tag{53}$$

$$\sigma_n = \sigma_n^I - k_n \Delta u_n^p \, . \tag{54}$$

We should note that necessary to modify elasticity matrix when the behavior of joint is plastic. The modified elasticity matrix is follow

$$D^{ep} = D^e - \frac{(D^e \cdot \frac{\partial g}{\partial \sigma_n})(D^e \cdot \frac{\partial f}{\partial \sigma_n})^T}{(\frac{\partial f}{\partial \sigma_n})^T \cdot D^e \cdot (\frac{\partial g}{\partial \sigma_n})} \tag{55}$$

where D^{ep} is modified elastic matrix, D^e is elasticity matrix, g is flow rule and f is failure criteria.

8. Numerical Examples

Example 1: Two block connected by a joint

Two block connected by joint subjected to **P=**5 KN load shown in Fig. (1) is studied. The data of joint and block, in this example are assumptive. The Mohr-Coulomb failure criterion is considered. The thickness of joint is 10cm and spacing of nodes in horizontal and vertical directions is 1m. The normal and shear stiffnesses of the joint are $K_s = K_n = 50$ MPa, and ν=0.3, E_r=20 GPa, Cohesion=10 Pa, Friction and Dilation angles=0°. The rock mass in plain strain state is analyzed. To compare Finite difference method and EFG results shown in Figs. (2) to (4). That good agreement exists between two set of results in elastoplastic state.

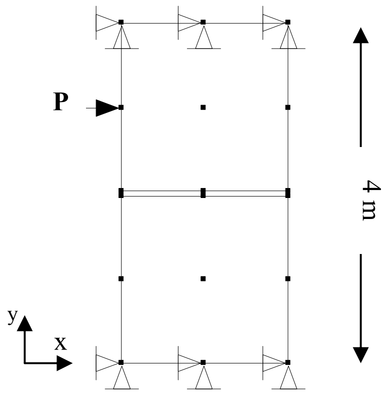

Fig. 1. The plastic analysis of simple jointed rock mass

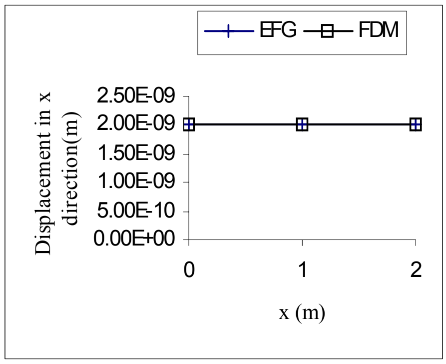

Fig. 2. Displacement in horizontal direction in y=1m

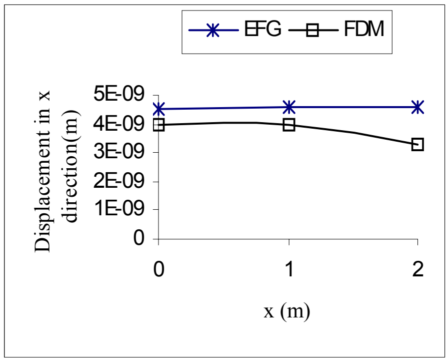

Fig. 3. Displacement in horizontal direction in y=2m

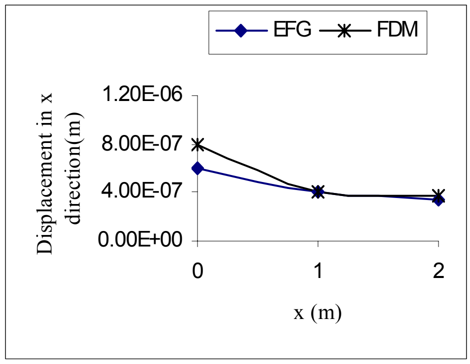

Fig. 4. Displacement in horizontal direction in y=3.1m

8. Conclusions

The elastic-plastic analysis of jointed rock mass using EFG method is new invention that in this paper is considered. The penalty method is used for essential boundary condition and to using of penalty method leads to accuracy results. Moving least squares approximation has used to construct shape function. To using of meshless methods in jointed rock mass can to avoid mesh-based difficulties such as remeshing, bad geometry of discontinuities and nulls and the time-consuming. The EFG method is an effective method for nonlinear problems. Using the EFG method for jointed rock mass in the elastic-plastic analysis with penalty method has shown good results.

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