The Mechanical Analysis of Jointed Rock Mass Using Meshless Method with Enforcing Modified Penalty Method

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Abstract

The methods based on meshes are not suitable for conventional computational methods such as finite element or finite difference when large deformations and large number of discontinuities are involved. The conventional process for dealing with moving discontinuities in these methods is to remesh the domain in each step of the process which leads in reduction of speed and degradation of accuracy and complexity of the calculations. To overcome these problems the meshless methods are introduced and applied which are found very suitable for problems with changeable geometry. There are a number of meshless methods, such as the Element Free Galerkin (EFG) method. In this paper EFG method is used for the mechanical analysis of intact rock mass and jointed rock mass, the modified penalty method is used for essential boundary condition and good results were obtained. The method based on meshes such as FDM[1] and FEM[2] have other difficulties such as bad geometry, because of null of domain or joint creation. Also the time of calculation for EFG method is less than that for mesh-based methods such as FDM. It is found that the present method is more suitable for modeling discontinuities of rock mass in two–dimensional and particular three-dimensional domains than conventional methods based on meshes.

Keyword: *Numerical Method, Rock Mechanics, Meshless, Element Free Galerkin.*

1- Introduction

Mostly the behavior of jointed rock mass is controlled by the discontinuities. The crack propagation and large deformation in jointed rock mass is considered. We need to model the large deformation and crack propagation properly with arbitrary paths. Many numerical methods have been proposed for jointed rock mass such as Finite Element Method, Finite Difference Method, Joint Element, Boundary Element Method, Discrete Element Method, Discontinuous Deformation Analysis, Rigid Finite Element Method,

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[1].Finite Difference Method
[2].Finite Element Method.

etc. The above methods are based on meshes and require the remeshing of the domain in each step of the evolution for change of geometry. Consequently mesh generation in each step of the evolution lead to degradation of accuracy and complexity in the computer program and it is a time consuming task. Thus the methods based on meshes are not suitable to the problems associated with large deformation and crack propagation that require the continuous remeshing of the domain. Although several strategies have been developed to maintain a reasonable mesh shape, such as the Arbitrary Lagrangian – Eulerian (ALE) method, extra computational effort and difficulties are also introduced [1]. Over the past three decades, many researchers have come to realize that so called meshless methods can be developed that eliminate the meshes and their difficulties. Fries and Matthies [2] published a special issue on meshless in July 2004 that is classification and overview of meshfree methods. They introduced different types of meshless methods such as Smooth Particle Hydrodynamics (SPH), Diffuse Element Method (DEM), Element Free Galerkin (EFG), Least Squares Meshfree Method (LSMM), Meshfree Local Petrov Galerkin (MLPG), Local Boundary Integral Equation (LBIE), Partition of Unity Methods (PUM), hp clouds, Natural Element Method (NEM), Meshless Finite Element Method (MFEM) and Reproducing Kernel Element Method (RKEM). In this paper EFG method is used for the mechanical analysis of jointed rock mass. Modified penalty method for enforcing the essential boundary conditions is applied. Example is shown good results and calculated time is less than FDM.

2- Element Free Galerkin Method (EFG)

Belytschko et al. [3] modified the constructing shape function for Diffuse Element Method (DEM) and they named the new method the Element Free Galerkin (EFG) method. The Moving Least Squares (MLS) approximation procedure related to construct shape function for EFG method. The MLS approximation $u^h(\mathbf{x})$ is defined as below:

$$u^h(\mathbf{x})=\sum_{j=1}^{m} p_j(\mathbf{x})a_j(\mathbf{x})\equiv\mathbf{p}^T(\mathbf{x})\mathbf{a}(\mathbf{x}) \quad (1)$$

where $p_j(\mathbf{x})$ are monomials of basis function in the space coordinates

$$\mathbf{x}^T=[x,y,z] \quad (2)$$

$\boldsymbol{a}(\mathbf{x})$ is a vector of coefficients and is obtained at any point $\mathbf{x}$ by minimizing J.

$$J=\sum_{I=1}^{m} w(\mathbf{x}-\mathbf{x}_I)[\mathbf{p}^T(\mathbf{x}_I)\mathbf{a}(\mathbf{x})-u_I]^2 \quad (3)$$

Where J is a function of weighted residual and constructed using the approximated values of the field function.

The stationary of J in Eq. (3) with respect to **a(x)** leads to the linear relation between **a(x)** and **u.**

$$\mathbf{A}(\mathbf{x})\mathbf{a}(\mathbf{x})=\mathbf{B}(\mathbf{x})\mathbf{u} \quad (4)$$

or

$$\mathbf{a}(\mathbf{x})=\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u} \quad (5)$$

where

$$\mathbf{A(x)}=\sum_{I=1}^{n} w_I\mathbf{(x)p}^T\mathbf{(x_I)p(x_I)} \quad (6)$$

$$\mathbf{B(x)}=[w_1\mathbf{(x)p(x_1)}, w_2\mathbf{(x)p(x_2)}, \dots, w_n\mathbf{(x)p(x_n)}] \quad (7)$$

$$\mathbf{u}=[u_1, u_2, \dots, u_n] \quad (8)$$

Substituting the equation (5) into (1) leads to

$$u^h\mathbf{(x)}=\sum_{I=1}^{n}\sum_{j=1}^{m} p_j\mathbf{(x)(A^{-1}(x)B(x))}_{jI}\, u_I \quad (9)$$

$$u^h\mathbf{(x)}=\sum_{I=1}^{n} \Phi_I\mathbf{(x)}\, u_I \quad (10)$$

where the MLS shape function $\Phi_I\mathbf{(x)}$ is defined by

$$\Phi_I\mathbf{(x)}=\sum_{j=1}^{m} p_j\mathbf{(x)(A^{-1}(x)B(x))}_{jI}\equiv\mathbf{p}^T\mathbf{(x)A}^{-1}\mathbf{(x)B(x)} \quad (11)$$

The partial derivative of $\Phi_I\mathbf{(x)}$ can be obtained as follows:

$$\Phi_{I,i}=\sum_{j=1}^{m}\left\{p_{j,i}\mathbf{(A^{-1}B)}_{jI}+p_j\mathbf{A}^{-1}\mathbf{(B}_{,i}-A_{,i}\mathbf{A}^{-1}\mathbf{B)}_{jI}\right\} \quad (12)$$

The weight function $w_I\mathbf{(x)}$ in this paper is used as [4]:

$$w_I\mathbf{(x)}=\begin{cases} 1-6*s^2+8*s^3-3*s^4 & \text{for} \quad : d_I<=r \\ 0 & \text{for} \quad : d_I>r \end{cases} \quad (13)$$

where $s=d_I/r$, $d_I=||\mathbf{x-x_I}||$, r=influence domain.

3- Displacement, Strain and Stress of Blocks and Joints

The displacement of any point in the block is obtained by:

$$u^h\mathbf{(x)}=\Phi\mathbf{(x)u_I} \quad (14)$$

where $\Phi\mathbf{(x)}$ is the shape function and is defined as:

$$\Phi\mathbf{(x)}=\mathbf{p}^T\mathbf{(x)A}^{-1}\mathbf{(x)B(x)} \quad (15)$$

$\mathbf{u}_I$ is displacement vector of nodes in the influence domain **x**. The displacement vector of nodes is obtained from equilibrium equation. The equilibrium equation can be obtained from variational principles [5]. The functional of the total potential energy (Π) of a jointed rock mass is given by:

$$\Pi=\Pi_b+\Pi_j+\Pi_f+\Pi_p \quad (16)$$

Where Π_b is the elastic strain energy of block, Π_j is the elastic strain energy of joints. An equivalent approach to express the equilibrium of the body is to use the principle of virtual displacements [6]. The total internal virtual work is equal to the total external virtual work that in a system of joint and block are defined as internal elastic strain energy. Π_f is the potential energy of the body force f_e, Π_p is the potential energy of the concentrated force **p**. These are given by:

$$\Pi_b=\sum_e t_e\iint_{\Omega e}½\varepsilon^T\mathbf{D}_b\ \varepsilon\ d_xd_y=½\sum_e\mathbf{u}_b^T(t_e\iint_{\Omega e}\mathbf{B}_b^T\ \mathbf{D}_b\mathbf{B}_bd_xd_y)\mathbf{u}_b \quad (17)$$

$$\Pi_j=\sum_k t_k\int_{\beta k}½\delta^T\mathbf{D}_j\ \delta d_j=½\sum_k\mathbf{u}_j^T(t_k\int_{\beta k}\mathbf{B}_j^T\ \mathbf{D}_j\mathbf{B}_jd_s)\mathbf{u}_j \quad (18)$$

$$\Pi_f=-\sum_e\iint_{\Omega e}\mathbf{u}^T\mathbf{f}_e\ d_xd_y=-\sum_e\mathbf{u}_b^T(\iint_{\Omega e}\mathbf{N}^T\ \mathbf{f}_e\ d_xd_y) \quad (19)$$

$$\Pi_p=-\sum_m\mathbf{u}^T\mathbf{p}_m=-\sum_m\mathbf{u}_b^T(\mathbf{N}^T\ \mathbf{p}_m) \quad (20)$$

We can obtain equilibrium equation from the stationary of functional Π in (16). The equilibrium equation of jointed rock mass is given by:

$$\mathbf{KU}=\mathbf{P} \quad (21)$$

where

$$\mathbf{K}=\sum_e t_e\iint_{\Omega e}\mathbf{B}_b^T\ \mathbf{D}_b\mathbf{B}_bd_xd_y+\sum_k t_k\int_{\beta k}\mathbf{B}_j^T\ \mathbf{D}_j\mathbf{B}_jds \quad (22)$$

$$\mathbf{P}=\sum_e\iint_{\Omega e}\mathbf{N}^T\ \mathbf{f}_e\ d_xd_y+\sum_m\mathbf{N}^T\mathbf{p}_m \quad (23)$$

$$\mathbf{U}=[u_1,v_1,u_2,v_2,\ldots,u_n,v_n]^T \quad (24)$$

Where t_e, t_k are the thickness of the block e and the joint k, respectively, n is the total number of nodes in the problem domain, **U** is the displacement vector related to the total number of nodes in the problem domain.
The displacement of any point into problem domain is obtained by:

$$\mathbf{u}=\mathbf{N}.\mathbf{u}_b \quad (25)$$

where

$$\mathbf{N}=\begin{bmatrix}\Phi_1(\mathbf{x}),0,\Phi_2(\mathbf{x}),0,\ldots,\Phi_n(\mathbf{x}),0\\ 0,\Phi_1(\mathbf{x}),0,\Phi_2(\mathbf{x}),0,\ldots,\Phi_n(\mathbf{x})\end{bmatrix} \quad (26)$$

and $\mathbf{u}_b$ is the displacement vector of influence domain **x. **The strain and stress at any point **x** are given by:

$$\boldsymbol{\varepsilon}=\mathbf{B}_b\mathbf{u}_b \quad (27)$$

$$\boldsymbol{\sigma}=\mathbf{D}_b\mathbf{B}_b\mathbf{u}_b \quad (28)$$

where

$$
\mathbf{B}_b = \begin{bmatrix} \Phi_{1,x}(\mathbf{x}) & , 0, & \Phi_{2,x}(\mathbf{x}), & 0, \dots, & \Phi_{n,x}(\mathbf{x}), & 0 \\ 0 & , \Phi_{1,y}(\mathbf{x}) & , 0, & \Phi_{2,y}(\mathbf{x}) & , 0, \dots, & \Phi_{n,y}(\mathbf{x}) \\ \Phi_{1,y}(\mathbf{x}), \Phi_{1,x}(\mathbf{x}) & & , \dots, & & \Phi_{n,y}(\mathbf{x}), & \Phi_{n,x}(\mathbf{x}) \end{bmatrix} \quad (29)
$$

$$
\mathbf{D}_b = (E/(1-\upsilon^2)) \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix} \quad \text{for plain stress state} \quad (30)
$$

$$
\mathbf{D}_b = (E/(1-2\upsilon)(1+\upsilon)) \begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & (1-2\upsilon)/2 \end{bmatrix} \quad \text{for plain strain state} \quad (31)
$$

and

$$
\boldsymbol{\varepsilon} = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}]^T \quad (32)
$$

$$
\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_{xy}]^T \quad (33)
$$

We can write:

$$
\mathbf{K} = \mathbf{K_b} + \mathbf{K_j} \quad (34)
$$

Where $\mathbf{K_b}$ and $\mathbf{K_j}$ are stiffness matrices for block and joint respectively. The moment matrix of joint can be defined as:

$$
\mathbf{B_j} = [\bar{0}, \dots, -\mathbf{LN_i}, \bar{0}, \dots, \mathbf{LN_j}, \bar{0}, \dots] \quad (35)
$$

Where $\mathbf{L}$ is the transform matrix and $\mathbf{N_i}$, $\mathbf{N_j}$ are the shape functions matrices of the i and j surfaces of joint. The elasticity matrix $\mathbf{D_j}$ of joint can be defined as:

$$
\mathbf{D_j} = \begin{bmatrix} k_s & 0 \\ 0 & k_n \end{bmatrix} \quad (36)
$$

The relative displacement vector δ of the joint is given as follows

$$
\delta = [\delta_n, \ \delta_s] = \tilde{u}_j - \tilde{u}_i = \mathbf{B_j u_j}
$$

where $\tilde{u}_i$ and $\tilde{u}_j$ are the displacement vectors of points i and j from the surface of joint, respectively.
Also $\mathbf{u_j}$ is:

$$
\mathbf{u_j} = [u_i^T, \ u_j^T]^T \quad (37)
$$

where u_i and u_j are displacements of point i and point j on the joint surfaces, respectively The strain vector of joint can be defined as :

$$\begin{bmatrix}\varepsilon_n \\ \gamma_{ns}\end{bmatrix} = \delta/h=(1/h)\mathbf{B_j u_j} \quad (38)$$

where ε_n is the normal strain and γ_{ns} is the shear strain. The stress vector of joint is:

$$\begin{bmatrix}\sigma_n \\ \tau_s\end{bmatrix} = \mathbf{D_j}\,\delta \quad (39)$$

Where σ_n is the normal stress and τ_s is the shear stress.

4- Numerical Example

Example 1: A jointed rock mass subjected to horizontal force is shown in Fig. (1). The length of rock mass equals to 8m and its high equals to 6m. The joint properties are K_n=K_s=.5e8 (pa/m) and rock property are ν=.3, E=2e7 pa. The number of mesh for any block is (2*8) and the total number of nodes is 81. This problem is solved in plane strain state. The number of quadrature point in each cell was 10*10(only for integration, cell are need), penalty factor was 1e5 and weight function is Quarticspline, influence radius of any node was 1.42. This problem is compared with FDM. The calculation time EFG is very much less than FDM, however results are close to each other. The results are shown in Fig. (2) to (4).

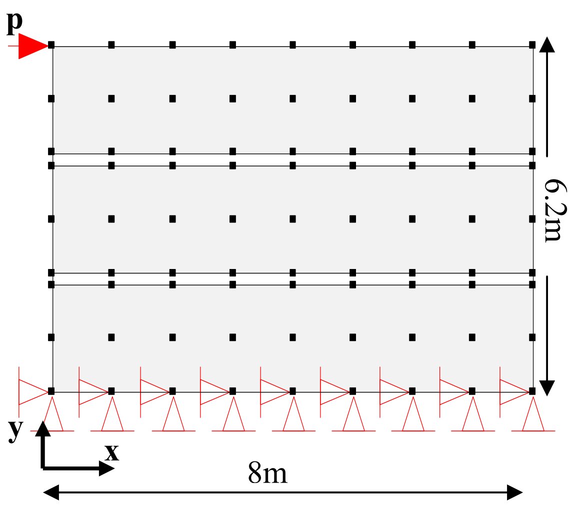

Fig (1): Three blocks that is connected with two joints

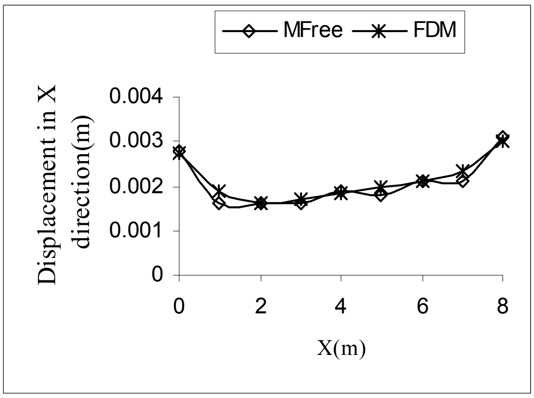

Fig (2): Displacement in x direction in y=1

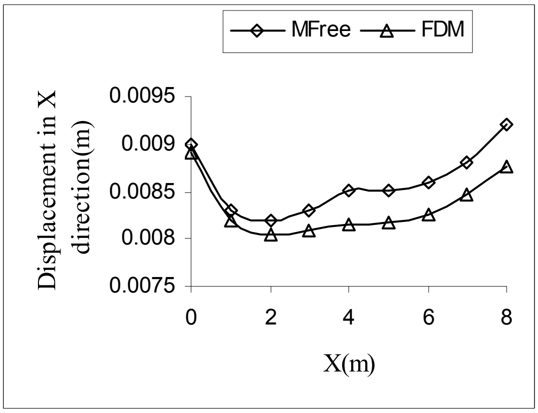

Fig (3): Displacement in x direction in y=3.1

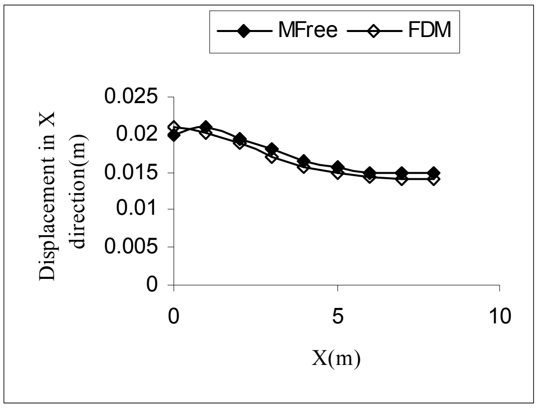

Fig (4): Displacement in x direction in y=5.2

Example 2: In this example is considered a jointed rock mass (Fig. (5)) subjected to the gravity load and a concentrated load P at point A [5]. The shear and normal stiffnesses for the vertical joints are 438.8 (horizontal=438.8) GPa/m and 395.9 (horizontal=531.9) GPa/m respectively. The elastic modulus of rock block is 89GPa and Poisson's ratio is 0.26. The specific gravity of the rock masses is 16KPa/m. The cohesion C of the joints is 20 KPa and friction and dilation angle 0˚. According to the plasticity theory the structure will be in a plastic limit state if the P reaches 2C. The relation between the horizontal displacement and the load P at point A is shown in table 1, that the load P exceeds 40 KN the rock mass will collapse and this coincides good with the plasticity theory.

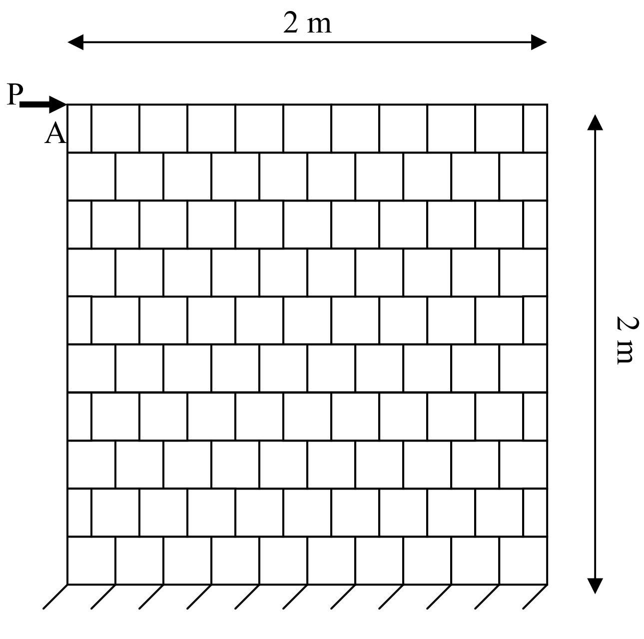

Fig (5): A jointed rock mass subjected to gravity and P load

Table (1): Displacement respect to P load at point A

Load P (KN)	5	10	15	20	25	30	35	40
Displacement at point A (*10^$^{-5}$m)	0.02	0.04	0.08	0.125	0.18	0.26	0.344	collapse

5- Conclusion

The methods based on meshes for large deformations and large number of discontinuities problems leads in reduction of speed and degradation of accuracy and complexity of the calculations. To overcome these problems the meshless methods are introduced and applied which are found very suitable for problems with change of geometry such as jointed rock mass problems. Using the element free Galerkin method for jointed rock mass in the elastic state and the elsto-plastic state showed good result.

6- References

[1] Noh,WF. CEL., "A Time- Dependent Two – Space – Dimensional Coupled Eulerian-Lagrangian Code",In Methods in Computational physics, Vol.3, Alder B, Fernbach S., and Rotenberg M. (eds). Academic Press:New York, 1964.

[2] Fries, T.-P., and Matthieș H.-G., "Classification and Overview of Meshfree Methods", July, 2004, Braunschweig Institut fűr Wissenschaftliches Rechnen Technische Universität Braunschweig.

[3] Belytschko T., and Lu YY., and Gu L., "Element-Free Galerkin Method", International Journal for Numerical Methods in Engineering, 1994, pp. 229 – 256.

[4] Liu G. R., "Mesh Free Methods: Moving beyond the Finite Element Method", CRC Press, 2002, Florida.

[5] Zhang X., and Lu MW., "Block-Interfaces Model for Nonlinear Numerical Simulations of rock Structures", International Journal of Rock Mechanics and Mining Sciences, 35(7), 1998, pp. 983-990.

[6] Bathe K. J., "Finite Element Procedures in Engineering Analysis", Prentice-Hall International, 1982, London.