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UNDRAINED BEARING CAPACITY OF CLAY BY ZERO EXTENSION LINE

CAPACITE PORTANTE NON-DRAINE PAR LIGNE D'EXTENSION ZERO

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SYNOPSIS: The method of zero extension line which has been used by the authors in predicting static and dynamic earth pressures and bearing capacities of sandy soils is extended to undrained behavior of clay soils. The shear strain of clay is related to cohesion and the bearing capacity is found to be a function of footing settlement. The ultimate bearing capacity prediction by zero extension line method approaches the well known Prandtl's solution.

INTRODUCTION

The zero extension line field (network of lines with zero elongation) was first introduced by Rosco (1970) and later used by James et al (1971) for predicting strain pattern behind retaining walls. The authors have used the method basically for sandy soils and have been able to predict static and dynamic earth pressure coefficients and bearing capacities. The method was further developed for predicting load displacement behavior of retaining walls and footings.

In this paper the zero extension line method is applied to undrained behavior of clayey soils with shear strength, q_u, equal to 2C (C is the cohesion of soil) and with angle of internal friction equal to zero. It is shown that both the stress characteristics and the characteristics of strain field (zero extension line field) coincide for undrained behavior of clay. General characteristics equation is developed.

The settlement of the footing is applied to the zero extension line to predict shear strain field. A relation between undrained strength and shear strain for clayey soils is then used to predict the developed cohesion under the footing as a function of settlement of the footing. Iterations are carried out until convergence is achieved.

It is shown that as the settlement increases, the pressure approaches the Prandtl's solution. Finally graphs are presented showing pressure distribution under the footing as function of settlement. A simplified procedure is presented for evaluation of pressure under the footing as function of settlement using the undrained strength as function of shear strain of clayey soil.

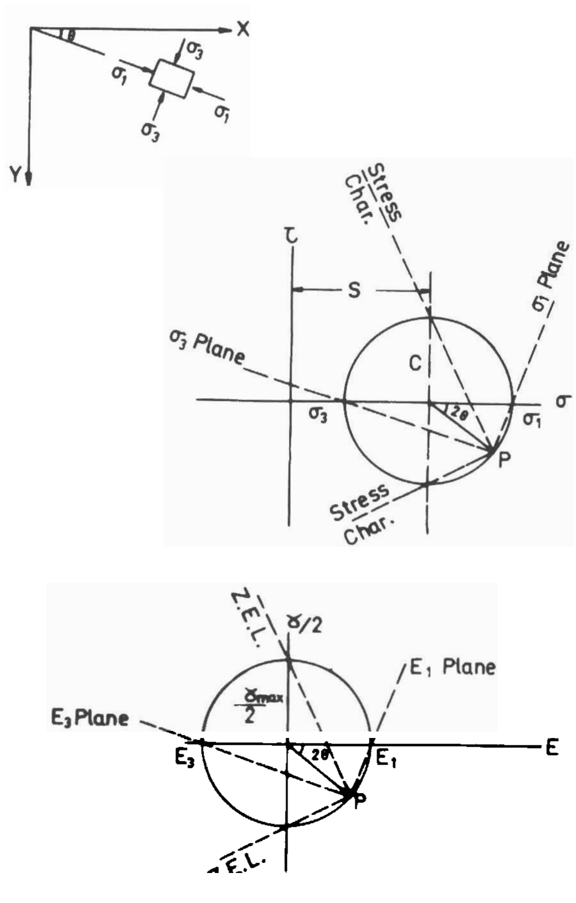

Fig. 1. Mohr circles of stress and strain

THEORY

Consider the Mohr circles of stress and strain in Fig. 1. If S is the length of origin to center of Mohr circle and C is mobilized cohesion, the equations of equilibrium are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad (1)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \Gamma \quad (2)$$

where σ_x, σ_y and τ_{xy} are stresses and Γ is the unit weight. If θ is the angle that direction of σ_1 makes with x-axis, then,

$$\sigma_x = S + C \cos 2\theta \quad (3)$$

$$\sigma_y = S - C \cos 2\theta \quad (4)$$

$$\tau_{xy} = C \sin 2\theta \quad (5)$$

The equations of equilibrium can be evaluated along two characteristics directions. On the direction:

$$\frac{dy}{dx} = \tan(\theta + \frac{\pi}{4})$$

we have

$$dS + 2Cd\theta = \Gamma dy - \frac{\partial C}{\partial x} dy + \frac{\partial C}{\partial y} dx \quad (6)$$

and on the direction:

$$\frac{dy}{dx} = \tan(\theta - \frac{\pi}{4})$$

we can write

$$dS - 2Cd\theta = \Gamma dy + \frac{\partial C}{\partial x} dy - \frac{\partial C}{\partial y} dx \quad (7)$$

These equations make possible the evaluation of S, θ, x and y on intersection of two characteristics if the values of the parameters at A and B (two points on the characteristics) are known (Fig. 2).

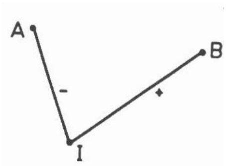

Fig. 2. Characteristics and intersection

Similarly using strain equations and assuming that principal directions of stress and strain coincide, the equations of displacements u and v (displacements along x and y axes respectively) on zero extension line direction:

$$\frac{dy}{dx} = \tan(\theta + \frac{\pi}{4})$$

is

$$du\, dx + dv\, dy = 0 \quad (8)$$

and on direction:

$$\frac{dy}{dx} = \tan(\theta - \frac{\pi}{4})$$

is

$$du\, dx + dv\, dy = 0 \quad (9)$$

The above equations signify that zero extension line network is like a network of connected linkages without any change of length along these linkages. Thus if u and v are known at A and B, u and v at intersection point I can be evaluated.

The method of evaluation of stress characteristics and displacement characteristics has been fully explained in previous works and will not be discussed here. In summary having the stresses in Rankine zone, the stresses in Goursat radial zone can be evaluated and eventually the stresses under the footing are obtained (Fig.3).

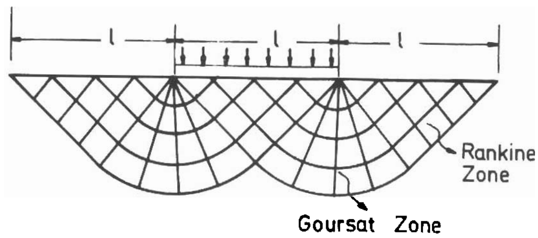

Fig. 3. Zero extension line network

Now the zero extension line network coincides with stress characteristics for undrained deformation of clay, therefore if settlement of footing is applied, the displacements can be evaluated at all points of zero extension line and eventually the shear strain γ can be calculated. Now using the relation between mobilized C and shear strain γ such as $C=f(\gamma)$ which is the case for undrained behavior of clay, the mobilized C can be determined. This process is iterated until convergence is achieved.

RESULTS

The relation between C and γ is assumed to be as shown in Fig.4. It can be seen that if $\gamma < \gamma_{cr}$:

$$C = C_0 + \frac{C_{max} - C_0}{\gamma_{cr}} \gamma \quad (10)$$

and otherwise:

$$C = C_{max} \quad (11)$$

This means that the undrained strength test can be approximated by Fig. 4. Of course, any relation between C and γ can be used. For the present case the initial cohesion is chosen as 100 kpa, the final cohesion as 200 kpa and it is assumed that the final cohesion will be developed at γ equal to 0.5. Fig. 5 shows the zero extension line network for the settlements of 0.00, 0.10 and 0.30 meters for 1 meter footing width.

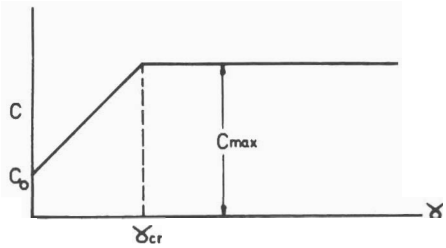

Fig. 4. Relation between mobilized C and γ

Fig. 6 shows the variation of pressure developed under the footing as function of settlement of footing. It is seen that at the beginning (settlement = 0) the bearing capacity is fully compatible with the Prandtl's solution:

$$p = 5.14C_0 + q \qquad (12)$$

where q is the surcharge. Also, at large settlements:

$$p = 5.14C_{max} + q \qquad (13)$$

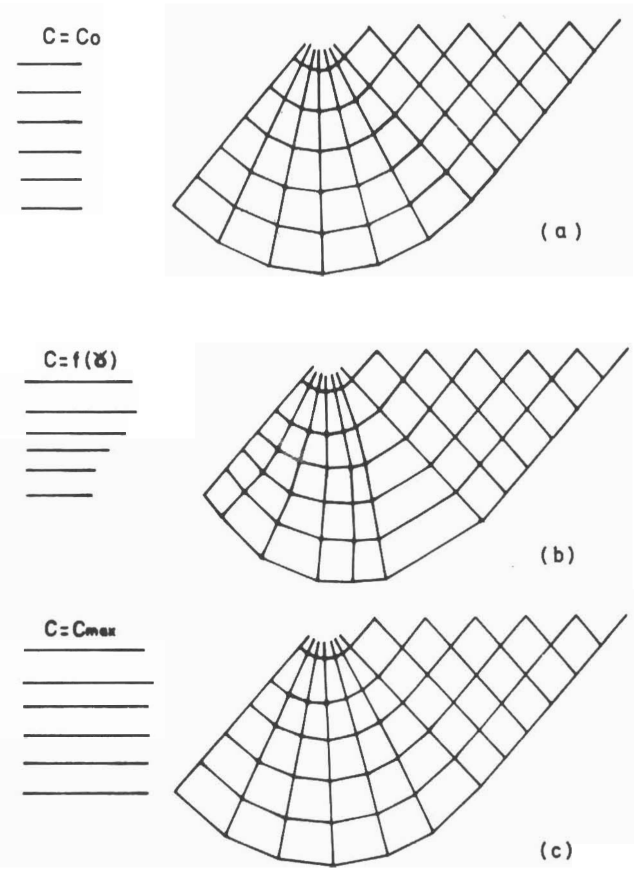

Fig. 5. Zero extension line networks for settlements: (a) 0.0, (b) 10 and (c) 30 cms

However in between, the pressure under the footin becomes variable, being generally larger at the edges. This is in conformity with the well known result that for Clayey soils the pressure under the edges is usually larger than the center of the footing (contrary to that of sandy soils).

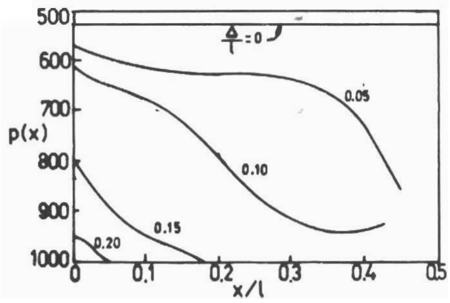

Fig. 6. Pressure under the footing as function of settlement

SIMPLIFIED ANALYSIS

If the radial zero extension line field is approximated to a circle, it can be shown that for the deformed zero extension line the angular shear strain is given as a function of r and Δ (Fig. 7): that is:

$$\gamma = \frac{\Delta}{r} \qquad (14)$$

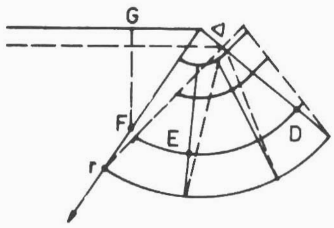

Fig. 7. Shear strain in the radial zore

Thus if it is assumed that for each circle such as DEF the value of shear strain is the same, then for the shear strain the mobilized cohesion can be evaluated and the pressure under the footing at point C above the point F can be determined. Thus, referring to Fig. 8, the pressure under the footing at a distance x from the center of the footing is:

$$p = p(x) = 5.14\ C(x) + q \qquad (15)$$

But,

$$\gamma = \frac{\Delta}{\ell/2 - x} \qquad (16)$$

where Δ is the footing settlement.

Thus if $\gamma < \gamma_{cr}$,

$$p(x) = 5.14\ (C_0 + \frac{C_{max} - C_0}{\gamma_{cr}} \cdot \frac{\Delta}{\ell/2 - x}) + q \qquad (17)$$

and if $\gamma > \gamma_{cr}$,

$$p = 5.14\ C_{max} + q \qquad (18)$$

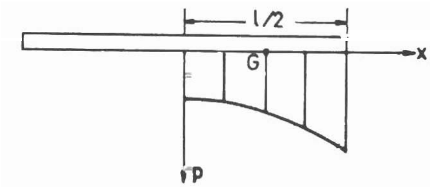

Fig 8 Pressure under the footing

The predicted pressure distribution is shown in Fig. 9 and is quite similar to Fig. 6.

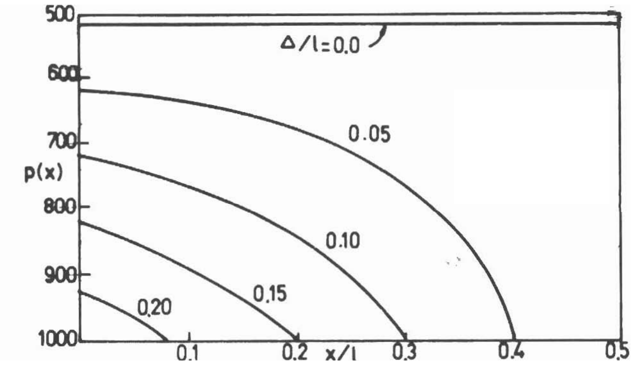

Fig. 9. Predicted pressure under the footing

CONCLUSIONS

Based on the above study, the following conclusions can be made,

1) Zero extension line method is capable of predicting bearing pressure under the footing for undrained clay soils.
2) The predicted pressure distribution is a function of settlement of the footing. At small settlements Prandtl's solution with initial developed cohesion of C_o is applicable and at large settlements, C_{max} is developed under the footing. The variation in between is shown in Fig. 6.
3) A simple formula can be used to evaluate pressure under the footing for clayey soils:

$$p = p(x) = 5.14\ C(x) + q$$

where

$$C(x) = f(\gamma)$$

and

$$\gamma = \frac{\Delta}{\ell/2 - x}$$

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