Prediction of Soil–Water Characteristic Curve Using Genetic Programming

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Abstract: In this technical note, a genetic programming (GP) approach is employed to predict the soil–water characteristic curve (SWCC) of soils. The GP model requires an input terminal set that consists of initial void ratio, initial gravimetric water content, logarithm of suction normalized with respect to atmospheric air pressure, clay content, and silt content. The output terminal set consists of the gravimetric water content corresponding to the assigned input suction. The function set includes operators such as plus, minus, product, division, and power. Results from pressure plate tests carried out on clay, silty clay, sandy loam, and loam compiled in the SoilVision software were adopted as a database for developing and validating the genetic model. For this purpose, and after data digitization, GP software (GPLAB) provided by MATLAB was employed for the analysis. Furthermore, GP simulations were compared with the experimental results as well as the models proposed by other investigators. This comparison indicated superior performance of the proposed model for predicting the SWCC.

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Introduction

Limitations in describing the mechanical behavior of unsaturated soils based on an effective stress equation, similar to the one proposed by Bishop and Donald (1961), has led to different approaches for modeling the observed behavior of these soils. Regardless of the approach adopted for modeling the soil behavior (effective stress based or approaches based on independent state variable), soil suction plays a major role. Therefore, soil–water characteristic curve (SWCC) is the backbone of any model for describing unsaturated soil behavior because it describes the variation of soil suction with changes in water content.

SWCC can be viewed as a continuous sigmoidal function describing the water storage capacity of a soil as it is subjected to various soil suctions. SWCC contains important information regarding the amount of water contained in the pores at a given soil suction and the pore size distribution corresponding to the stress state in the soil (Fredlund et al. 2002). Unsaturated soil behavior such as shear strength, volume change, diffusivity, and adsorption are related to the SWCC (Fredlund and Rahardjo 1993).

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Existing Methods for Determining Soil–Water Characteristic Curves

There are several methods available for obtaining the SWCC for a particular soil. SWCC may be determined directly or indirectly in the laboratory. Direct methods include pressure plate, buchner funnel, tensiometers, and pressure membranes. These methods measure the pore-water pressure in the soil or impose a known air pressure to soil and allow the water content to come to equilibrium with the imposed air pressure. Among these methods, conventional pressure plate (*ASTM D 6836*) is the most common method. Indirect methods include filter paper and heat dissipation sensors. These methods use measurements or indicators of water content or a physical property that is sensitive to changes in water content. However, these experiments are costly and time consuming. Therefore, several empirical methods have been proposed in the literature. These methods can be classified into four major groups described below:

1. In the first group, water contents at each suction value are correlated to specific soil properties such as D_{10} (sieve size for 10% passing) and porosity. This process generally requires a regression analysis followed by a curve fitting procedure. Among important contributions are the works of Hutson and Cass (1987), and Aubertin et al. (1998).
2. The second group includes methods that correlate parameters of an analytical equation with basic soil properties such as grain size distribution and dry density using a regression analysis. Among important contributions are the works of Cresswell and Paydar (1996) and Tomasella and Hodnett (1998).
3. The third group is based on physico-empirical modeling of SWCC. This approach converts the grain size distribution into a pore size distribution, which in turn is related to a distribution of water content and associated pore pressure. Among important contributions are the works of Fredlund et al. (1997) and Zapata et al. (2003).

4. Artificial intelligence methods such as neural network, genetic programming, and other machine learning methods fall into the fourth group. No significant attempt on this subject is cited in the literature.

The first two approaches are similar but they differ in the parameters that are correlated with soil properties. In this technical note, a genetic programming (GP) approach is proposed to estimate the SWCC using basic soil properties such as grain size distribution, initial void ratio, initial water content, as well as logarithm of suction normalized with respect to atmospheric air pressure.

Genetic Programming

GP, a branch of the genetic algorithm (GA) (Holland 1975), is a method for learning the most "fit" computer programs by means of artificial evolution. In other words, its behavior forms a metaphor of the processes of evolution in nature. GP, similar to GA, initializes a population that compounds the random members known as chromosomes (individual). Afterward, fitness of each chromosome is evaluated with respect to a target value. The principle of Darwinian natural selection is used to select and reproduce "fitter" programs. The main difference between GP and GA is the representation of the chromosomes and final solution. A genetic algorithm creates equal length strings of numbers (chromosomes) in the form of binary or real which represent the solution. However, GP creates equal or unequal length computer programs [a symbolic expression that consists of variables (terminal) and several mathematical operators (function)] in the LISP language or other computer languages as the solution. Therefore, unlike GA, in GP there is no need to define the form of the objective function a priori. In fact, it is the GP that determines not only the coefficients and parameters of the objective function, but also and more importantly, the form of the objective function itself. This is one of the advantages of GP as compared to GA. Although research on GP techniques dates back to the 1960s and 1970s, GP emerged as a distinct discipline presented by Koza (1990).

In brief, five stages are employed in GP to solve a problem.

1. **Initialize a population of programs**. Create a population of randomly generated programs in LISP language.
2. **Selection**. The randomly generated programs with the higher fitness will "win" and must be copied to the next generation. There are several different types of selection used in GP such as roulette-wheel selection, tournament, and ranking.
3. **Transform the winner programs**. The two winner programs (GP solution) are then copied and transformed probabilistically by: exchanging parts of the winner programs with each other to create two new programs (crossover) and randomly changing each of the winner programs to create new program. A function can only replace a function, a terminal can only replace a terminal, and an entire subtree can replace another subtree (mutation).
4. **Replace the "loser" programs**. Replace the "loser" programs in the population with the transformed "winner" programs. The winners of the selection remain in the population unchanged.
5. **Iterate until convergence**. Repeat steps 2–4 until a program is developed that predicts the behavior properly.

GP Modeling of SWCC

A GP software, GPLAB (Silva 2003) in conjunction with subroutines coded in MATLAB were used in this study. Five parameters, namely void ratio, initial water content, logarithm of suction normalized with respect to atmospheric air pressure, clay content, and silt content, were selected as the input terminals. The output terminal was the gravimetric water content corresponding to the assigned input suction. To find the optimum formulation five functions, namely plus, minus, product, division, and power, were used. A large number of generations were needed to find a formula with minimum error. First, the maximum depth of the tree and the length of the branch were assigned. With these constants, a large number of generations were required to minimize the error. These constants were changed and the program was executed to search for a formula with minimum error and as short as possible in length. The optimum GP structure had the following characteristics:

1. Selection Method: Selection is done by the Lexictour method. In this method, similar to the tournament approach, a random number of individuals are taken from the population and the best fit is chosen. The main difference is that if two individuals are equally fit, the shortest one (tree with less nodes) is chosen as the best.
2. Operations: The operations that were used in this study were crossover and mutation. They were selected by adopting a rule with a minimum probability of 0.1.
3. Fitness Function: The sum of absolute differences between the obtained and expected gravimetric water content for all sets of data in the database was adopted as a measure for fitness.
4. Population size: 100 members.
5. Maximum depth of tree representation allowed during generations: 7.
6. Total generations: 6,000.

Database

Results of the pressure plate tests performed on clay, silty clay, sandy loam, and loam soil reported by various researchers and compiled by *SoilVision (2002)* were adopted for the analysis. Table 1 indicates the range of basic soil properties employed for this study. This database consists of the results from 186 pressure plate tests together with their grain size distributions. Final suction values typically ranged from 800 to 1,700 kPa with few tests having suction values as large as 10^5 kPa. Pressure plate test results were then digitized to obtain the necessary database. For digitization, an increasing incremental value of suction was adopted. Hence, the suction value was doubled in each increment. Initial suction value was fixed at 0.2 kPa. The database thus de-

Table 1. Range of Basic Soil Properties for Samples from *SoilVision* (2002) Adopted for Developing GP Model

Property	Range
Void ratio	0.458–2.846
Suction (kPa)	0.2–104,857.6
Specific gravity	2.28–2.92
Water content (%)	0.18–98.27
Dry density (kg/m^3)	702–1,811
Initial water content (%)	17.34–105.41
Clay content (<0.005 mm)(%)	4.4–76.7
Silt content (0.005–0.075 mm)(%)	10.3–87.5
Sand content (>0.075 mm)(%)	0.1–55.3

veloped had a total of 2,694 patterns. For normalization each component of the data set was normalized to lie in an interval of [0,1] using a max-min approach.

Model Development and Validation

From the 186 pressure plate tests used in this study, 131 tests were selected randomly for developing the GP model and the remaining 55 tests were used to validate the GP. Hence, after digitization of the test results, from a total of 2,694 generated patterns, 1,894 patterns were used for model development, and the remaining 800 patterns were allocated for validation proposes.

The optimum GP program (optimum formulation) is obtained by evolving the programs toward the formulation with minimum error compared with the actual test results. In this process, soil parameters and suction values are assigned and the sum of absolute differences between the predicted water content and the actual water content was monitored. Iterations continued until this error measure did not decrease appreciably. The GP error dropped from 209 in the first generation to about 53 after 6,000 generations.

Results

After developing the model, the formula generated by the GP had the following form:

$$Y = 0.794(X_2 + 0.215)\{[(0.116^{X_3} \times X_4^{X_5})^{(X_1+0.234)} + (X_4^{0.368(X_5/X_4)}) \times (X_3^{X_1} - X_3)]X_4\}^{X_3^2} \quad (1)$$

This formula was then scaled based on initial water content to yield

$$\omega = X_2(Y/Y_0) \quad (2)$$

where Y= predicted water content; Y_0= predicted initial water content (at suction 0.2 kPa); X_1= initial void ratio; X_2= initial water content; X_3= log[suction(kPa)/p_a] where p_a= atmospheric pressure (taken as 100 kPa); X_4= clay content (%); X_5= silt content (%); and ω= adjusted water content.

Eqs. (1) and (2) were used to simulate the SWCCs of all 131 pressure plate tests in the modeling set and all 55 tests in the validation set. Figs. 1 and 2 compare predicted gravimetric water content with the actual data for modeling and validation, respectively. These figures show a good correlation between the predictions made using GP formulation and the actual data both for modeling and validation data.

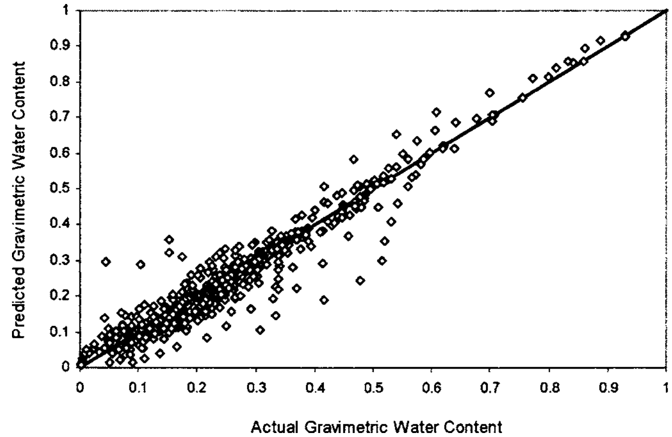

Fig. 2. Actual versus predicted GWC for validation data in GP (R^2=0.93)

The proposed model may be used to predict SWCC solely from basic soil parameters without resorting to a sophisticated experimental test. The procedure includes the following five steps:

1. Determine the predicted initial water content (Y_0) by inputting normalized soil parameters (X_1, X_2, X_4, and X_5) and choose an initial suction value of X_3=0.2 kPa;
2. Choose a series of suction pressures say 2,4,8,16,…, 1,500 kPa;
3. Determine the predicted water content (Y) corresponding to each suction value of step 2 using Eq. (1);
4. Determine the adjusted water content using Eq. (2); and
5. Obtain SWCC by drawing suction values from step 2 versus corresponding water content from step 4.

Estimation of SWCC using the proposed model and following the above mentioned procedure requires input parameters that may be determined easily using simple laboratory tests that may take only 1 or 2 days as opposed to the lengthy laboratory procedure (20–30 days) needed for SWCC determination.

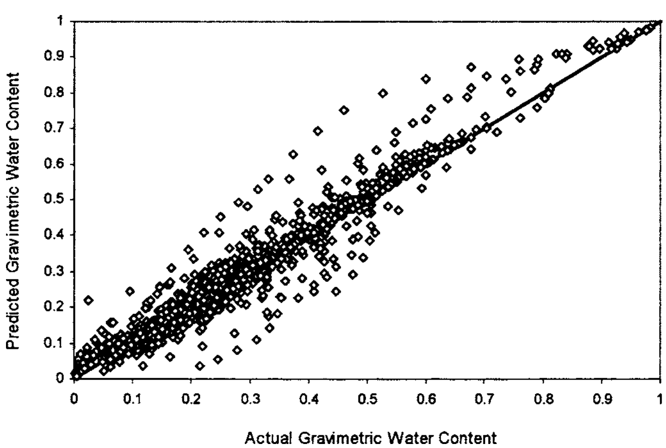

Fig. 1. Actual versus predicted GWC for modeling data in GP (R^2=0.94)

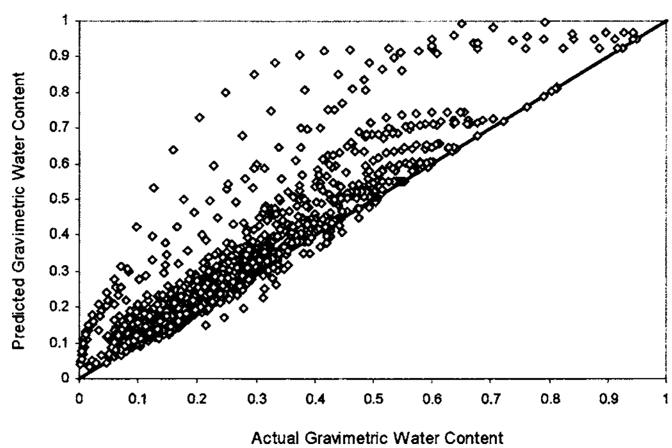

Fig. 3. Actual versus predicted GWC for modeling data using Fredlund et al. approach (R^2=0.85)

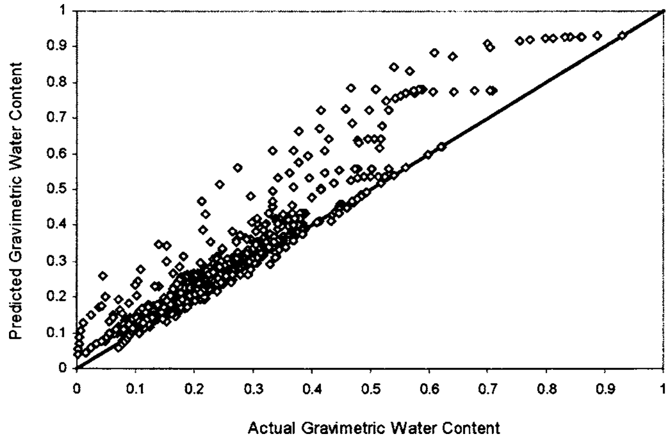

Fig. 4. Actual versus predicted GWC for validation data using Fredlund et al. approach (R^2=0.89)

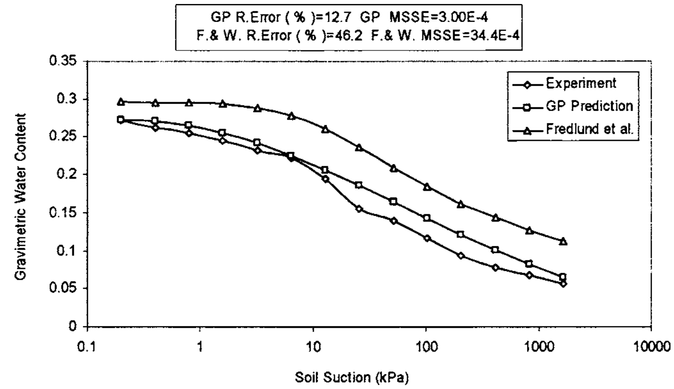

Fig. 5. Average simulation results among tests used for developing GP model (void ratio: 0.8; initial water content: 27.23%; clay content: 16.98%; silt content: 75.23%; dry density: 1,500 kg/m^3; specific gravity: 2.7)

Comparison with Previous Works

As stated before, a number of methods have been presented by various investigators for estimating SWCC. Among these methods, the approach presented by Fredlund et al. 1997 was considered to give a more reasonable estimate of the SWCC approach (Fredlund et al. 2002 and Zapata et al. 2003). Therefore, in this technical note, the proposed model is compared with the approach presented by Fredlund et al. (1997).

Figs. 3 and 4 compare predicted gravimetric water content by the Fredlund et al. (1997) approach versus actual data for the modeling and validation set, respectively. Table 2 presents the error in GP prediction compared with the aforementioned approach. In this table, the average relative error (ARE) and the mean sum squared of the error (MSSE) are defined by

$$\text{ARE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{A_i - P_i}{A_i}\right| \times 100 \tag{3}$$

$$\text{MSSE} = \frac{1}{N}\sum_{i=1}^{N}(A_i - P_i)^2 \tag{4}$$

R^2= correlation coefficient (square of the Pearson product moment correlation coefficient) where A_i= actual value for ith data; P_i= predicted output data for ith data; and N= total number of data available in the database

From Table 2 it can be concluded that the proposed GP method is capable of simulating SWCC more accurately compared to the conventional methods. In order to show the robustness of the proposed method, simulation results were also compared on a one-to-one basis. Fig. 5 shows SWCC for a specimen used in developing the model. In this figure, GP prediction, experimental data, and prediction based on the method suggested by Fredlund et al. (1997) are also shown. From this figure it may be concluded that the formulation has a good potential for predicting SWCC with reasonable accuracy. Similarly, Fig. 6 presents the prediction of GP for a typical test not used in developing the model (validation). From this figure, it may also be concluded that the proposed method is also capable of simulating new test results, though not as good as tests used in the modeling phase. The test results used to measure the performance of the proposed GP model correspond to average simulations among the modeling and validation data sets. These results indicate the robust feature of the GP to learn and predict the SWCC without making any assumption or simplifications a priori.

Table 2. Performance of Genetic Programming and Fredlund et al. (1997) Approach

	Modeling data			Validation data		
Method	ARE (%)	MSSE	R^2	ARE (%)	MSSE	R^2
GP	13.59	0.0016	0.94	16.69	0.0015	0.93
Fredlund et al.	34.73	0.0071	0.85	35.39	0.0047	0.89

Conclusion

A model based on GP was proposed to estimate the SWCC for soils. A database containing the results of pressure plate tests carried out on a wide variety of fine grained soils was employed to develop the model. Test results were then digitized and normalized to obtain the necessary database. During the first phase, the model was developed using the results from 131 pressure plate tests. In the second phase, it was validated using 55 additional test

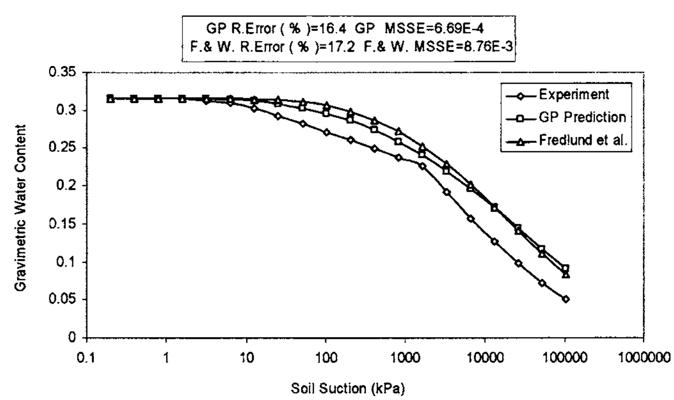

Fig. 6. Average simulation results among tests used for validation (void ratio: 0.852; initial water content: 31.57%; clay content: 70.33%; silt content: 23.7%; dry density: 1,457.7 kg/m^3; specific gravity, Gs: 2.7)

results that the model had not been exposed to during the first phase. The model prediction indicated a reasonable accuracy both for the results used in the first phase, as well as results in the validation phase. The model prediction had some discrepancies compared to the actual test data, however comparison of the results from the proposed model with conventional methods indicated its superior performance for prediction of SWCCs.

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