

**METHOD OF ZERO EXTENSION LINES IN  
STRESS STRAIN ANALYSIS OF GRANULAR MEDIA**

A. Sabzevari, Mechanical Engineering Department  
University of Puerto Rico, Mayaguez, P.R. 00708

A. Grahramani, Civil Engineering Department  
Shiraz University, Shiraz, Iran

The failure analysis for granular media has been traditionally based on an assumed simplified stress field without any reference to the deformation field. The medium has been assumed rigid plastic and the upper bound plastic theory is employed to determine the critical load. This simplified classical method does not always give satisfactory results.

To obtain the actual stress field in a granular medium, the deformation field must be first specified, and here is exactly where the present authors method of associated fields has become very useful [1,2,3]. To simplify the computational process of the method of associated fields a new scheme of computational stress-strain analysis named the method of zero extension lines [4,5,6] is developed. In the former method two fields of stress and displacement characteristics are constructed and analyzed separately whereas in the latter, only a simplified strain characteristic field is developed which is applicable to the circular and logarithmic spiral fields.

In the study of load-deformation behavior of soil-structure problems such as retaining walls and foundations, generalized zero extension line fields based on the following stress characteristic equations are developed along the zero extension lines;

$$dS - 2S \tan\phi d\theta = \gamma[-\tan\phi dx + dz] + S[\tan\phi d\phi - \frac{\partial\phi}{\partial z^+} \frac{dz^+}{\cos\phi}] \quad (1)$$

$$dS + 2S \tan\phi d\theta = \gamma[\tan\phi dx + dz] + S[\tan\phi d\phi - \frac{\partial\phi}{\partial z^-} \frac{dz^-}{\cos\phi}] \quad (2)$$

which are true on the characteristic directions

$$\frac{dz}{dx} = \tan(\theta - \mu) \quad \text{and} \quad (3)$$

$$\frac{dz}{dx} = \tan(\theta + \mu) \quad (4)$$

respectively. Allowing up to 5% of approximation, the above stress equations along the zero extension lines

$$\frac{dz}{dx} = \tan(\theta - \zeta) \quad (5)$$

$$\frac{dz}{dx} = \tan(\theta - \zeta) \quad (6)$$

are respectively expressed as:

$$dS - 2S \alpha \tan \phi d\theta = \gamma \beta [-\tan \phi dx + \alpha dz] + S [\tan \phi d\phi - \frac{\partial \phi}{\partial d^-} \frac{dd^-}{\cos \phi}] \quad (7)$$

$$dS + 2S \alpha \tan \phi d\theta = \gamma \beta [\tan \phi dx + \alpha dz] + S [\tan \phi d\phi - \frac{\partial \phi}{\partial d^+} \frac{dd^+}{\cos \phi}] \quad (8)$$

In above equations  $\phi$  denotes angle of friction,  $\nu$  is angle of dilation  $\zeta = \pi/4 - \nu/2$  and  $\mu = \pi/4 - \phi/2$ .  $\theta$  is angle between major principal stress and  $x$  axes and  $S$  designates the average of major and minor principal stresses, i.e.  $S = (\sigma_1 + \sigma_3)/2$ . The terms  $dx^-$  and  $dx^+$  are elements of length along negative and positive stress characteristics. Similarly  $dd^-$  and  $dd^+$  are for displacement characteristics.  $\gamma$  is the soil unit weight.

Solutions of eqs. (5) through (8) result in values of stresses along the zero extension lines directly. Having the deformation equation

$$u dx + v dz = 0 \quad (9)$$

where  $u$  and  $v$  are displacements along  $x$  and  $z$  axes the complete load deformation is then obtained by using the zero extension line fields. It should be noted that in this method no zero extension line field is assumed a priori but it is developed by using equations 5 through 8.

In conclusion the results of soil structure problems computed by this method is compared with those obtained through the method of associated fields and close similarity is achieved.

## REFERENCES

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