Pacific Conference on Earthquake Engineering

Australia, 20-22 November 1995

SEISMIC BEARING CAPACITY FACTORS BY ZERO EXTENSION LINE METHOD

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SUMMARY

The seismic bearing capacity of soils is evaluated by the method of zero extension line. The inclination of footing reaction as well as surcharge due to horizontal acceleration is considered. The equilibrium equation along the zero extension line is used to evaluate N_c, N_q and N_γ, the cohesion, surcharge and gravity bearing capacity factors. It is found that due to seismic acceleration, N_c, N_q and N_γ are reduced and bearing capacity is further reduced due to inertia of the soil mass.

INTRODUCTION

The seismic bearing capacity of foundations have been studied recently by Sarma and Iossifelis (1990) and Richards, Elms and Budhu (1993). The method of zero extension line (lines with linear strain equal to zero), first proposed by Roscoe (1970) and later used by James and Bransby (1971) for predicting the strain pattern behind retaining walls was extended by Habibagahi and Ghahramani (1970) for static earth pressure. The extension to dynamic earth pressure was carried out by Ghahramani and Clemence (1990). Bearing capacity was dealt with by the zero extension line method by Behpoor and Ghahramani (1987) and (1994). Anvar and Ghahramani (1995) treated active dynamic pressure on retaining walls. The method was applied to seismic bearing capacity of clay soils by Ghahramani and Berrill (1995). It is the purpose of the present paper to study the seismic bearing capacity of general soils having both cohesion and friction. Both the surcharge q and the footing pressure p have horizontal components during seismic loading. The tangent of inclination angle equals k_c which is the horizontal fraction of gravity acceleration due to seismic loading. The effect of horizontal inertia component has also been included in the analysis.

THEORY

The theoretical development of the method of zero extension line has been dealt with in the references cited. It is shown that in limiting equilibrium there are two zero extension line (ZEL) directions which make angle $\xi = \frac{\pi}{4} - \frac{v}{2}$ with the direction of σ_1 axis of major compressive stress (Fig. 1). Angle v is the angle of dilation of soil.

The two directions are given by the following equations:

$$\text{on the "plus" line} \qquad \frac{dz}{dx} = \tan(\theta + \xi) \qquad (1)$$

$$\text{on the "minus" line} \qquad \frac{dz}{dx} = \tan(\theta - \xi) \qquad (2)$$

where θ is the angle between the direction of σ_1 and the x axis.

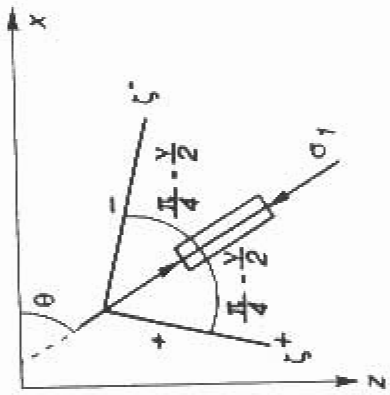

Figure 1   Zero extension line directions

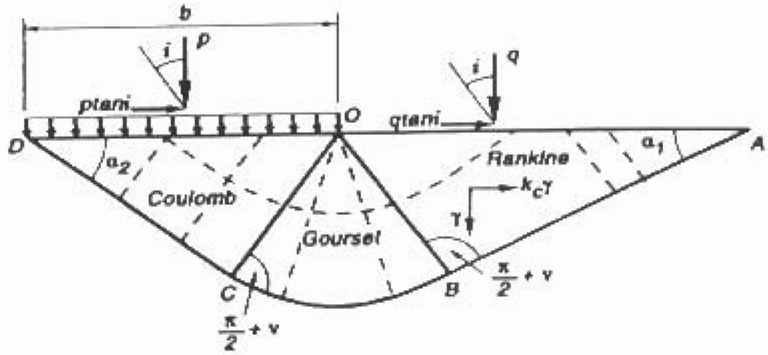

Figure 2. Seismic bearing capacity ZEL field

The equilibrium equations along the two zero extension line directions are

On the plus line

$$
\begin{aligned}
du + 2\,(u \tan\phi + c)\left(\hat{\alpha}\, d\theta + \hat{\eta}\,\frac{\partial\theta}{\partial\zeta^-}\, d\zeta\right) = \\
- X\hat{\beta}\,[\tan\phi\, dz - \hat{\alpha}\, dx] + Z\hat{\beta}\,[\tan\phi\, dx + \hat{\alpha}\, dz] \\
+ (u - c\tan\phi)\left(\tan\phi\, d\phi - \frac{1}{\cos\phi}\,\frac{\partial\phi}{\partial\zeta^-}\, d\zeta\right) \\
+ \left(\tan\phi\, dc - \frac{1}{\cos\phi}\,\frac{\partial c}{\partial\zeta^-}\, d\zeta\right)
\end{aligned}
\tag{3}
$$

On the minus line

$$
\begin{aligned}
du - 2\,(u \tan\phi + c)\left(\hat{\alpha}\, d\theta + \hat{\eta}\,\frac{\partial\theta}{\partial\zeta^+}\, d\zeta\right) = \\
X\hat{\beta}\,[\tan\phi\, dz + \hat{\alpha}\, dx] - Z\hat{\beta}\,[\tan\phi\, dx - \hat{\alpha}\, dz] \\
+ (u - c\tan\phi)\left(\tan\phi\, d\phi - \frac{1}{\cos\phi}\,\frac{\partial\phi}{\partial\zeta^+}\, d\zeta\right) \\
+ \left(\tan\phi\, dc - \frac{1}{\cos\phi}\,\frac{\partial c}{\partial\zeta^+}\, d\zeta\right)
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
\hat{\alpha} = \frac{1 - \sin\phi \sin\nu}{\cos\phi \cos\nu} \quad \hat{\eta} = \frac{\sin\phi - \sin\nu}{\cos\phi \cos\nu} \quad \hat{\beta} = \frac{\cos\nu}{\cos\phi} \\
\lambda = \frac{\sin\phi - \sin\nu}{\cos\phi} \quad u = \frac{\sigma_x + \sigma_z}{2}
\end{aligned}
\tag{5}
$$

and X and Z are the horizontal and vertical body forces.

It should be noted that for any function f, along the plus direction

$$\tan\phi \, d\phi - \frac{1}{\cos\phi} \frac{\partial f}{\partial \zeta^-} d\zeta = \lambda \, df + \hat{\beta} \left(\frac{\partial f}{\partial z} dx - \frac{\partial f}{\partial x} dz \right) \tag{6}$$

on a minus line

$$\tan\phi \, d\phi - \frac{1}{\cos\phi} \frac{\partial f}{\partial \zeta^+} d\zeta = \lambda \, df + \hat{\beta} \left(\frac{\partial f}{\partial x} dz - \frac{\partial f}{\partial z} dx \right) \tag{7}$$

During earthquake loading, the surcharge q and footing pressure p act with inclination i (Fig. 2) such that tan i = k_c, which is the horizontal fraction of gravity acting due to the seismic loading.

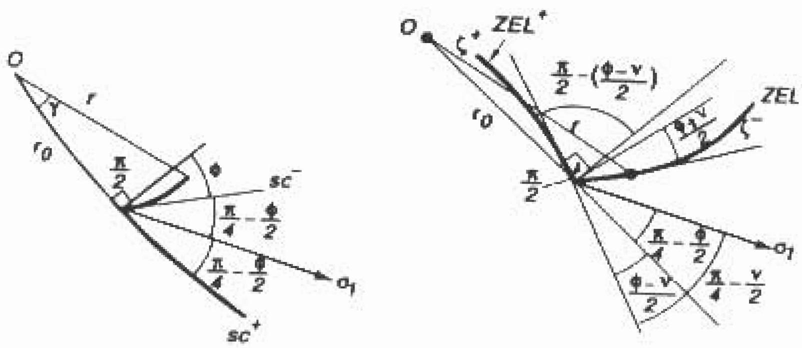

Figure 3. Stress characteristics and ZEL network near singularity

The zero extension line (ZEL) field is composed of three fields: a Rankine zone AOB, a radial Goursat shear zone OBC and Coulomb zone OCD under the footing with width equal to b. It should be noted that the zero extension line network is straight in Rankine and Coulomb zones and in Goursat zones it is composed of straight radial lines and spirals with angle ν for minus direction.

The equations of equilibrium should be integrated along ABCD to get bearing capacity. The result is presented in the classical form as

$$p = c\,N_c + q\,N_q + \frac{1}{2}\gamma b\,N_\gamma \tag{8}$$

For a constant φ, c soil it is simpler to define a reduced mean stress $u_r = u + c \cot\phi$. Then the equilibrium equation along the ZEL in the (-) direction becomes:

$$du_r - 2\tan\phi \, u_r \left(\hat{\alpha} \, d\theta + \hat{\eta} \frac{\partial \theta}{\partial \zeta^+} d\zeta \right) = \tag{9}$$

$$X\hat{\beta} \; (\tan\phi \, dz + \hat{\alpha} \, dx) - Z\hat{\beta} \; (\tan\phi \, dx - \hat{\alpha} \, dz)$$

In this equation $Z = \gamma$, and $X = k_c\gamma$ for the seismic case, where γ is unit weight.

In the following analysis as is customary for bearing capacity calculations, N_c and N_q are evaluated first using a soil with $\gamma = 0$, then N_γ is evaluated using a soil with $c = 0$.

EVALUATION OF N_C AND N_Q

For the soil with $\gamma = 0$ the equation of equilibrium reduces to

$$du_r - 2\tan\phi \, u_r \left(\hat{\alpha} \, d\theta + \hat{\eta} \frac{\partial \theta}{\partial \zeta^-} d\zeta \right) = 0 \tag{10}$$

Now $\dfrac{\partial\theta}{\partial\zeta^+}$ is non zero through the singularity at point O.

Through the singularity, the stress characteristics (Fig. 3) are composed of straight radial lines and logarithmic spirals with spiral angle = ϕ for <u>minus</u> stress characteristics.

The Mohr circle for surcharge q is shown in Figure 4 and the circle for bearing pressure p is shown in Figure 5.

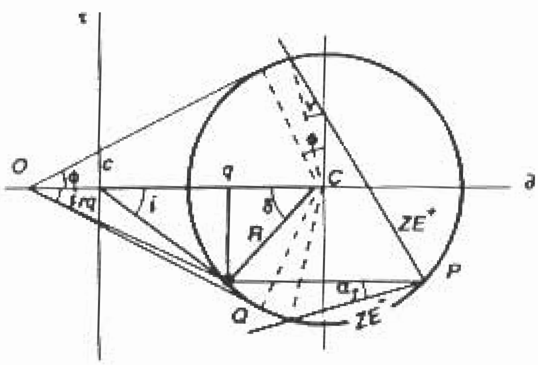

Figure 4. Surcharge Mohr circle

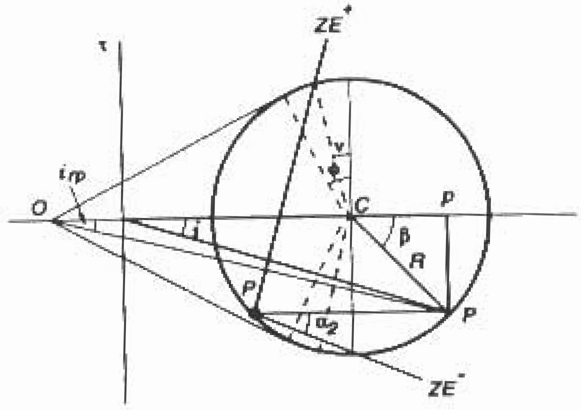

Figure 5. Bearing pressure Mohr circle

However the ZEL is composed of two sets of logarithmic spirals: for the minus direction, the spiral angle is $(\phi + v)/2$; and for the plus direction, the spiral angle is $\pi/2 - (\phi - v)/2$. Thus these equations are valid:

(−) stress characteric $\qquad r = r_0\, e^{\tan\phi\, \alpha}$

(−) ZEL $\qquad r = r_0\, e^{\tan\left(\frac{\phi + v}{2}\right)\alpha}$ (11)

(+) ZEL $\qquad r = r_0\, e^{-\tan\left(\frac{\pi}{2} - \frac{\phi - v}{2}\right)\alpha}$

where r_o is the original radial distance measured from O; r is the radial distance to a point on the spiral making an angle α with r_o. Then we get

(−) sc $\qquad dr = r_0\, e^{\tan\phi\,\alpha} \tan\phi \, d\alpha = r \tan\phi \, d\alpha$

(−) ZEL $\qquad dr = r_0\, e^{\tan\frac{(\phi + v)}{2}\alpha} \tan\dfrac{\phi + v}{2}\, d\alpha = r\tan\left(\dfrac{\phi + v}{2}\right) d\alpha$

(+) ZEL $\qquad dr = -r_0\, e^{-\tan\left(\frac{\pi}{2} - \frac{\phi - v}{2}\right)\alpha} \tan\left(\dfrac{\pi}{2} - \dfrac{\phi - v}{2}\right) d\alpha$ (12)

$$= -r\tan\left(\frac{\pi}{2} - \frac{\phi - v}{2}\right) d\alpha$$

Now since

$$d\zeta^+ = r\, d\alpha / \cos\left(\frac{\pi}{2} - \frac{\phi - v}{2}\right) \quad \text{and} \quad d\zeta^- = r\, d\alpha / \cos\left(\frac{\phi + v}{2}\right) \tag{13}$$

then

$$\frac{\partial\theta}{\partial\zeta^+} = \frac{1}{r}\cos\left(\frac{\phi + v}{2}\right) \qquad \text{and} \qquad \frac{\partial\theta}{\partial\zeta^-} = \frac{-1}{r}\cos\left(\frac{\pi}{2} - \frac{\phi - v}{2}\right) \tag{14}$$

Therefore

$$\hat{\alpha}\ \frac{\partial\theta}{\partial\zeta^-} + \hat{\eta}\ \frac{\partial\theta}{\partial\zeta^+} = \frac{\partial\theta}{\partial\zeta^-}\left(\hat{\alpha} + \hat{\eta}\ \frac{\partial\theta}{\partial\zeta^+} \Big/ \frac{\partial\theta}{\partial\zeta^-}\right)$$

$$= \frac{\partial\theta}{\partial\zeta^-}\left[\frac{1 - \sin\phi\sin\nu}{\cos\phi\cos\nu} - \frac{\sin\phi - \sin\nu}{\cos\phi\cos\nu}\ \frac{\sin\left(\frac{\phi-\nu}{2}\right)}{\cos\left(\frac{\phi+\nu}{2}\right)}\right]$$

Agebraic manipulation shows that the quantity in the [ ] is equal to unity. Thus in the vicinity of the singularity

$$\hat{\alpha}\ \frac{\partial\theta}{\partial\zeta^-} + \hat{\eta}\ \frac{\partial\theta}{\partial\zeta^+} = \frac{\partial\theta}{\partial\zeta^-} \tag{15}$$

and the equation of equilibrium becomes

$$du_r - 2\tan\phi\ u_r\ d\theta = 0$$

This is the famous equation of Sokolovksi for change of u_r along singularity.

From the figures it is clear that theta for surcharge

$$\theta_q = \frac{\delta}{2} \quad \text{and} \quad \theta_p = \frac{\pi}{2} - \frac{\beta}{2} \tag{17}$$

$$\alpha_1 = \frac{\pi}{4} - \frac{\nu}{2} - \frac{\delta}{2} \qquad \alpha_2 = \frac{\pi}{4} + \frac{\nu}{2} - \frac{\beta}{2} \tag{18}$$

$$\tan i_{rq} = \tan i\ \frac{q}{q + c\ \cot\phi} \qquad \tan i_{rp} = \tan i\ \frac{p}{p + c\ \cot\phi}$$

If we call $c/p = c_{op}$ and $c/q = c_{oq}$ then

$$i_{rq} = \tan^{-1}\left[1 \Big/ \left(1 + \frac{c_{oq}}{\tan\phi}\right)\right] \quad \text{and} \quad i_{rp} = \tan^{-1}\left[1 \Big/ \left(1 + \frac{c_{op}}{\tan\phi}\right)\right]$$

Also, in triangle OCQ, OC = $u_q = q + c\cot\phi$ and $\frac{R}{OC} = \frac{\sin i_{rq}}{\sin(\delta + i_{rq})} = \frac{1}{\sin\phi}$

and in triangle OCP in Fig. 5 we have OC = $u_p = p + c\cot\phi$.

$$\frac{R}{OC} = \frac{\sin i_{rp}}{\sin(\beta - i_{rp})} = \frac{1}{\sin\phi}$$

which results in

$$\delta = \sin^{-1}\left(\frac{\sin i_{rq}}{\sin\phi}\right) - i_{rq} \qquad \beta = \sin^{-1}\left(\frac{\sin i_{rp}}{\sin\phi}\right) + i_{rp} \tag{20}$$

From equation 16 it is seen that

$$\frac{u_{rp}}{u_{rq}} = e^{2\tan\phi(\theta p - \theta q)} = e^{2\tan\phi\left(\frac{\pi-\beta-\delta}{2}\right)} \tag{21}$$

If we let $\quad p_r = p + c\cot\phi$ and $q_r = q + \cot\phi$ (22)

From triangles OCQ and OCP (similar to Sokoloski (1965)) it is found that

$$\begin{aligned} q_r &= \cos i_{rq}\left(\cos i_{rq} - \sqrt{\sin^2\phi - \sin^2 i_{rq}}\right) u_{rq} \\ p_r &= \cos i_{rq}\left(\cos i_{rq} + \sqrt{\sin^2\phi - \sin^2 i_{rq}}\right) u_{rp} \end{aligned} \tag{23}$$

If N_q is defined as

$$N_q = \frac{p_r}{q_r} = \frac{p + c\cot\phi}{q + c\cot\phi} \tag{24}$$

then using eq. (23) and (21), N_q is evaluated as

$$N_q = \frac{\cos i_{rp}\left(\cos i_{rp} + \sqrt{\sin^2\phi - \sin^2 i_{rp}}\right)}{\cos i_{rq}\left(\cos i_{rq} - \sqrt{\sin^2\phi - \sin^2 i_{rq}}\right)} \; e^{\tan\phi(\pi - \delta - \beta)} \tag{25}$$

If the usual form of $p = cN_c + qN_q$ is used, then it is clear from (24) that

$$p = N_q\, q + c\cot\phi\,(N_q - 1) \tag{26}$$

Thus

$$N_c = \cot\phi\,(N_q - 1) \tag{27}$$

Since N_q is a function of c/q it has to be evaluated by trial and error. Fig. 6 shows N_q plotted as a function of k_c, the horizontal acceleration ratio, for ϕ from 1 to 45 degree for the case of c/q = 0 which corresponds to c = 0.0. The radial lines in Fig. 6 and 7 show the ratio of mobilisation of shear strength for surcharge region which varies betwen 0 to 1. (The ratio of shear mobilisation of surcharge is defined as $(q \tan i)/(c + q\tan\phi)$. Fig. 8 gives N_q vs friction angle in degrees for various values of k_c up to 0.5g.

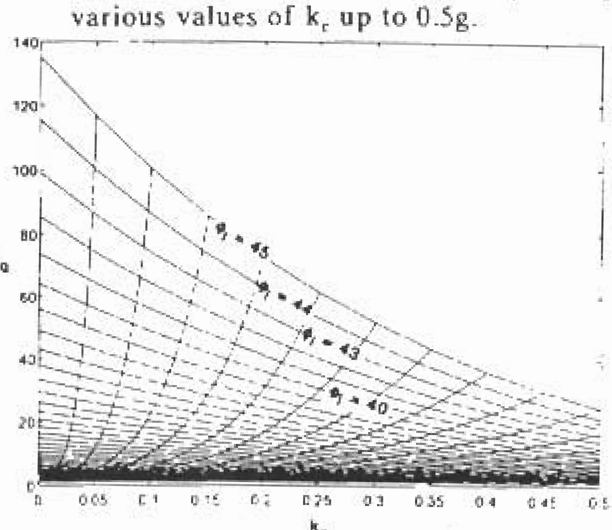

Figure 6. Bearing capacity factor N_q for c = 0

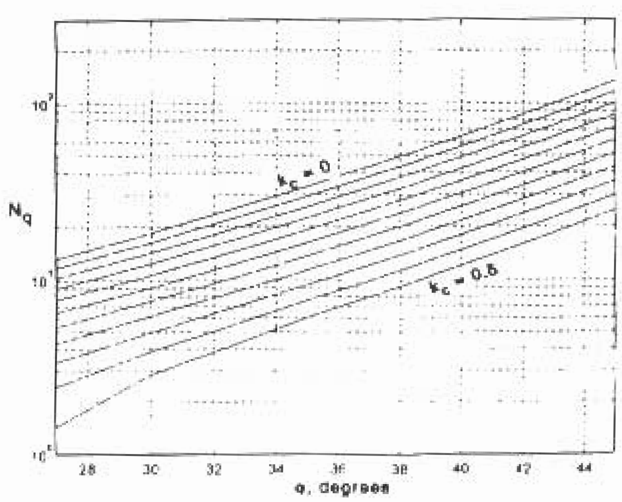

Figure 7. Bearing capacity factor N_q for c = 0, plotted as a function of ϕ

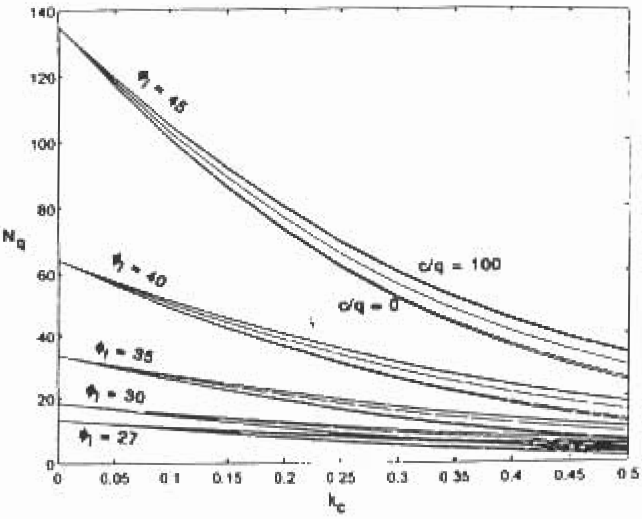

Figure 8. N_q for c/q = 0, 0.1, 1, 10, 100

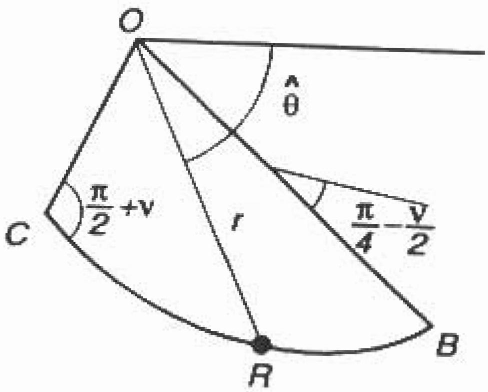

Figure 9. Gourset zone

It is found that for c = 0, N_q agrees with values obtained by Meyerhoff (1953), Sokolovski (1965) and Sarma and Iossifelis (1990). However for c/q different from zero, different results are obtained. Fig. 8 presents N_q in linear scale, for ϕ = 45, 40, 35, 30, 27 degrees and for c/q = 0, 0.1, 1, 10, 100. It is clear that at larger k_c values, an increase of c/q (larger cohesion or smaller surcharge) results in increases in N_q values. Curves for N_c are not presented because equation (27) makes the evaluation easy.

EVALUATION OF N_γ

For evaluation of N_γ, integration of the equilibrium equation is carried out along ABCD. Since radial zero extension lines are straight $\partial\theta/\partial\zeta^+$ is equal to zero. If footing width is denoted by b, the geometric relations below follow (refer to Fig. 2):

$$OC = \frac{b\sin\alpha_2}{\cos\nu} \qquad x_c = \frac{-b\sin\alpha_2}{\cos\nu}\sin(\alpha_2-\nu) \quad z_c = \frac{b\sin\alpha_2}{\cos\nu}\cos(\alpha_2-\nu) \tag{28}$$

$$OB = OC\ e^{\tan\nu(\alpha_1-\alpha_2)} = \frac{b\sin\alpha_2}{\cos\nu}\ e^{\tan\nu\left(\frac{\pi}{2}-\frac{\delta}{2}-\frac{\beta}{2}\right)} \tag{29}$$

$$x_B = \sin(\alpha_1+\nu)\ \frac{b\sin\alpha_2}{\cos\nu}\ e^{\tan\nu\left(\frac{\pi}{2}-\frac{\delta}{2}-\frac{\beta}{2}\right)}$$

$$z_B = \cos(\alpha_1+\nu)\ \frac{b\sin\alpha_2}{\cos\nu}\ e^{\tan\nu\left(\frac{\pi}{2}-\frac{\delta}{2}-\frac{\beta}{2}\right)} \tag{30}$$

$$x_A = OA = \frac{b\sin\alpha_2}{\sin\alpha_1}\ e^{\tan\nu\left(\frac{\pi}{2}-\frac{\delta}{2}-\frac{\beta}{2}\right)}, \qquad z_A = 0 \tag{31}$$

Remembering that Z = γ and X = $k_c\gamma$, then integration along AB, with $d\theta$ = 0, yields

$$u_{rB} = u_{rA} + X\hat{\beta}\left[\tan\phi(z_B-z_A) + \hat{\alpha}(x_B-x_A)\right] - Z\hat{\beta}\left[\tan\phi(x_B-x_A) - \hat{\alpha}(z_B-z_A)\right] \tag{32}$$

where u_{rA} and u_{rB} are reduced u at A and B. Integration from B to C in the Goursat zone is carried out by using geometric relations from Fig. 9.

$$\text{Let} \quad \hat{\theta} = \theta + \frac{\pi}{4} - \frac{\nu}{2}$$

$$\text{Then} \quad OR = OB\ e^{-\tan\nu(\hat{\theta}_A-\hat{\theta}_B)} = OB\ e^{-\tan\nu(\hat{\theta}_R-\hat{\theta}_B)} \tag{33}$$

$$\text{and} \quad x = OR\cos(\hat{\theta}) = OB\ e^{-\tan v(\theta - \theta_B)}\cos\hat{\theta}$$
$$\text{and} \quad z = OR\sin(\hat{\theta}) = OB\ e^{-\tan v(\theta - \theta_B)}\sin\hat{\theta} \tag{34}$$

Furthermore

$$dx = OB\ e^{-\tan v(\theta - \theta_B)}\left[-\tan v\cos\hat{\theta} - \sin\hat{\theta}\right]d\theta$$
$$\text{or} \qquad dx = -OB\ e^{-\tan v(\theta - \theta_B)}\sin(\theta + \mu)/\cos v\ d\theta \tag{35}$$
$$\text{Similarly,} \quad dz = OB\ e^{-\tan v(\theta - \theta_B)}\cos(\theta + \mu)/\cos v\ d\theta$$

where $\mu = \pi/4 + v/2$.

By using equation (9) we get

$$du_r - 2\hat{\alpha}\tan\phi\, u_r\, d\theta = \frac{\hat{\beta}}{\cos v}\left\{X\left[\tan\phi\cos(\theta + \mu) - \hat{\alpha}\sin(\theta + \mu\right]\right.$$
$$\left. + Z\left[\tan\phi\sin(\theta + \mu) + \hat{\alpha}\cos(\theta + \mu)\right]\right\} OB\ e^{-\tan v(\theta - \theta_B)}\ d\theta \tag{36}$$

Noting that

$$\int e^{-k\alpha}\cos(\alpha + \beta)\, d\alpha = \cos\lambda\sin(\alpha + \beta - \lambda)\, e^{-k\alpha}$$
$$\int e^{-k\alpha}\sin(\alpha + \beta)\, d\alpha = -\cos\lambda\cos(\alpha + \beta - \lambda)\, e^{-k\alpha} \tag{37}$$

where $\tan\lambda = k$, after some manipulation and setting $k = 2\hat{\alpha}\tan\phi + \tan v$ we get

$$u_{rC} = u_{rB}\, e^{2\tan\phi\alpha_j(\theta_C - \theta_B)} + OB\,\hat{\beta}\frac{\cos\lambda}{\cos v}\, e^{\tan\theta_B - 2\tan\phi\hat{\alpha}\theta_C}$$
$$\left\{e^{-k\theta_C}\left[X(\tan\phi\sin\xi_C + \hat{\alpha}\cos\xi_C) - Z(\tan\phi\cos\xi_C - \hat{\alpha}\sin\xi_C)\right]\right.$$
$$\left. -e^{k\theta_B}\left[(\tan\phi\sin\xi_B + \hat{\alpha}\cos\xi_B) - Z(\tan\phi\cos\xi_B - \hat{\alpha}\sin\xi_B)\right]\right\} \tag{38}$$

$$\text{where} \quad \xi_B = \theta_B + \pi/4 + v/2 - \lambda \quad \text{and} \quad \xi_C = \theta_C + \pi/4 + v/2 - \lambda$$

Furthermore integration along CD will result in

$$u_{rD} = u_{rc} + X\hat{\beta}\left[\tan\phi(z_D - z_C) + \hat{\alpha}(x_D - x_c)\right]$$
$$- Z\,\hat{\beta}\left[\tan\phi(x_D - x_c) - \hat{\alpha}(z_D - z_c)\right] \tag{39}$$

To evaluate N_γ, we set $u_{rA} = 0$, $c = 0$, $X = k_c\gamma$ and $Z = \gamma$. Because the pressure distribution under the footing is linear, we set p equal to the average pressure in the bearing capacity formula $p = cN_c + qN_q + 0.5\gamma b\, N_\gamma$ and use equation (23) to find N_γ. The result of computing N_γ is shown in Fig. 10 for ϕ = 45, 40, 35, 30, 27 degrees and for k_c ranging from zero to 0.5. The results are lower than those of Sarma and Iossifelis although with the similar trend, and almost identical to Sokolowski (1965) for $k_c = 0$.

Fig. 11 shows $\frac{1}{2}N_\gamma$ plotted as a function of ϕ, logarithmic scale. It is found that the ratio of seismic bearing capacity factor N_γ to the static bearing capacity factor depends mainly on k_c and does not depend strongly on friction angle ϕ. This trend is shown in Fig. 12. This reduction curve can be used to calculate the seismic bearing capacity factor N_γ if static bearing capacity factor N_γ is evaluated by other methods. This trend compares favourably with the ratio evaluated by Richards, Elms and Budhu (1993) in their Fig. 5(a).

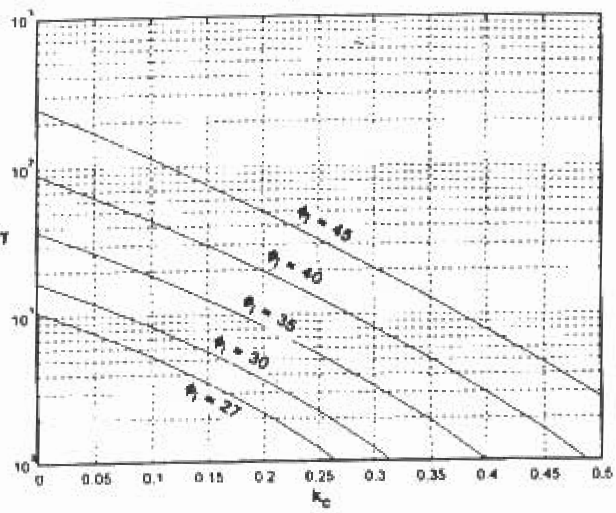

Figure 10. Bearing capacity factor N_γ plotted against seismic coefficient k_c

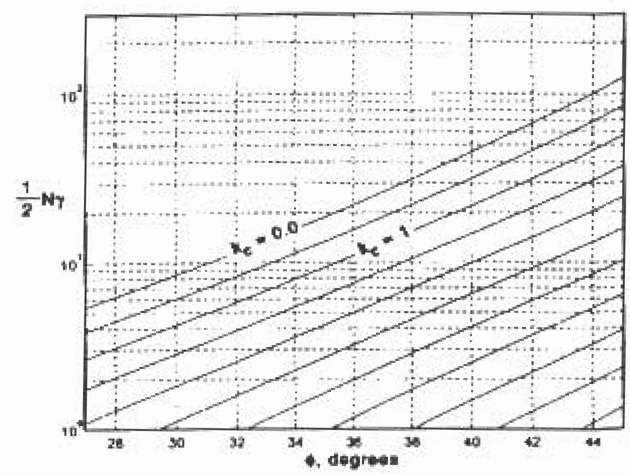

Figure 11. Bearing capacity factor .5 N_γ

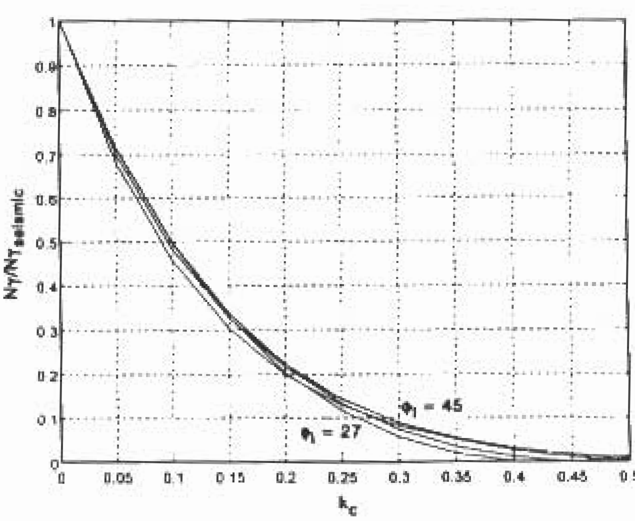

Figure 12. Seismic to static bearing capacity ratios

CONCLUSIONS

Based on the work presented in the paper the following conclusions can be made:

1. The method of zero extension line is capable of predicting seismic bearing capacity factors.

2. The cohesion, surcharge and gravity bearing capacity factors, N_c, N_q and N_γ are reduced due to seismic acceleration, and inertia forces within the soil.

3. For c=0, N_q agrees with values obtained by Meyerhof (1953), Sokolovski (1965) and Sarma and Iossifelis (1990). However, for c/q different from zero, different values are obtained. In all cases $N_c = \cot\phi\,(N_q - 1)$.

4. N_γ values are lower than Sarma and Iossifelis (1990) but have similar trends. N_γ values are identical to Sokolowski for $k_c = 0$.

5. The ratio of seismic bearing capacity factor N_γ over the static bearing capacity factor depends on k_c and is not sensitive to value of ϕ. This ratio by zero extension line theory compares favourably with the results of Richards, Elms and Budhu (1993).

ACKNOWLEDGEMENT

The authors acknowledge the support of the University of Canterbury and Shiraz University.

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