Prediction of the bearing capacity and load–displacement behavior of shallow foundations by the stress-level-based ZEL method

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Abstract The theory of Zero Extension Lines (ZEL), based on the solution of soil plasticity equations along ZEL directions, has wide applications in determination of the bearing capacity and load–displacement behavior of foundations and retaining walls. It is known that soil behavior and shear strength parameters are stress level dependent. In fact, a dense soil presenting a dilative behavior under low stress levels may show a contractive behavior under higher levels of stress. On the other hand, foundation size has a significant effect on the level of imposed stress on subsoil elements. In this work, the ZEL method is employed to consider the stress level dependency of soil strength in the bearing capacity computation and load–displacement behavior of foundations. A computer code is developed to solve ZEL equations in MATLAB. The results obtained by this numerical model have been then compared with experimental tests and those obtained by other methods.

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1. Introduction

Among several different approaches in determination of the bearing capacity of shallow foundations, the famous triple-N formula of Terzaghi [1] has been generally employed in the past decades, and can be written as follows:

\[ q_{ult} = cN_c + qN_q + 0.5\gamma BN_p. \]  

(1)

In this equation, \( q_{ult} \) is the ultimate bearing capacity, \( c \) is cohesion, \( q \) is surcharge pressure, \( B \) is foundation width, \( \gamma \) is soil density and \( N_i \) coefficients are bearing capacity factors, which are functions of the soil friction angle. The third term has been known as the main contributor to the bearing capacity of shallow foundations on cohesionless soils. However, unlike the first two factors, i.e. \( N_c \) and \( N_q \), the third factor is the most challengeable. There are several suggested values for the third factor by different authors [1–7]. Although all these methods are, generally, based on a limit analysis solution, there are differences between their assumptions for boundary conditions and consideration of the soil weight effect. The third bearing capacity factor, \( N_p \), has been computed by taking several assumptions into account. For example, Terzaghi [1] assumed that the components of the bearing capacity equation can be safely superposed. Bolton and Lau [5] performed a study on the effect of surcharge pressure on computed \( N_p \), and presented a dimensionless factor, \( \Omega \), defined as the ratio of the surcharge pressure, \( q \), to \( \gamma B \). They stated that if this factor is equal to, or less than, 1.0, the effect of surcharge pressure leads to less than 20% error in calculation of the bearing capacity factor, \( N_p \), which seems to be acceptable for practical purposes. Beside these assumptions, almost all conventional methods assume a constant field of soil friction angle to compute the bearing capacity factors.

Considering the bearing capacity equation, the third term suggests an increasing tendency in bearing capacity with an increase in foundation size. However, data from De Beer [8], Bolton and Lau [9] and Clark [10] show that the bearing capacity of shallow foundations does not increase with size linearly, and the bearing capacity factor, \( N_p \), decreases by increasing foundation size. Recently, other investigations showed similar results [11–15]. On the other hand, among some researchers, Fellenius and Altaee [16] by inspecting the load–displacement behavior of prototype foundations, showed that for settlements...
even beyond 10% of the footing width (or diameter), the foundation behavior is not that of an approaching or reaching ultimate failure mode.

Such observations are resulted from an important effect called the scale effect, which can be related to the stress level experienced by soil.

1.1. Zero extension lines method

The Zero Extension Lines (ZEL) theory, which was introduced by Roscoe [17], has been widely developed in past decades. This is a powerful numerical method with applications to a wide range of problems in soil mechanics dealing with stress and strain analyses. James and Bransby [18] used them for strain and deformation prediction behind a model retaining wall. Habibagahi and Ghahramani [19] and Ghahramani and Clemence [20] calculated soil pressures by considering the force-equilibrium of soil elements between the zero extension lines. Jahanandish et al. [21,22] presented the methodology for finding the load–deflection behavior of foundation and retaining walls on the basis of the ZEL theory. The general forms of these lines were considered in this methodology and the variations of soil shear strength parameters, i.e. cohesion and friction angle, with any intersecting point. In soil mechanics, $\nu$ is known to be the angle of soil dilation. If the co-axiality (similarity between the stress characteristics and the ZEL) has already been used in obtaining the load–deflection behavior of structures in contact with soil. Therefore, the ZEL method can be reasonably used for consideration of the stress level effect on soil shear strength properties and consequently foundation behavior.

![Figure 1: ZEL directions: (a) minor and major principal strains and (b) directions of the stress characteristics and the ZEL](image)

Based on the Jahanandish derivation [24], the final form of equilibrium-yield equations along the ZEL directions for the more general case of axial symmetry is:

Along the minus ($-$) ZEL:

$$
\frac{\partial T}{\partial \sigma_{\xi}} d\epsilon_{\xi} - \frac{2T}{\cos \nu} \left( \frac{\partial \psi}{\partial \epsilon_{\xi}} d\epsilon_{\xi} - \frac{\partial \psi}{\partial \epsilon_{\xi}} d\epsilon_{\xi} \right) = \left[ f_{s} \cos (\psi - \xi) + f_{z} \sin (\psi - \xi) \right] d\xi
$$

Along the plus ($+\ +\ +\ +\ +$) ZEL:

$$
\frac{\partial T}{\partial \sigma_{\xi}} d\epsilon_{\xi} + \frac{2T}{\cos \nu} \left( \frac{\partial \psi}{\partial \epsilon_{\xi}} d\epsilon_{\xi} + \frac{\partial \psi}{\partial \epsilon_{\xi}} d\epsilon_{\xi} \right) = \left[ f_{s} \cos (\psi + \xi) + f_{z} \sin (\psi + \xi) \right] d\xi.
$$

In these equations, $T$ is the radius of the Mohr circle for stress, $S$ is equal to $(\sigma_{1} + \sigma_{2})/2$ and $f_{s}$ and $f_{z}$ are expressed by [24]:

$$
f_{s} = X - \frac{nT}{r} (1 + \cos 2\psi)
$$

$$
f_{z} = Z - \frac{nT}{r} \sin 2\psi
$$

where $n$ is an integer equal to 1 for the axi-symmetric problems and 0 for the plane strain case. As mentioned, these equations are more general, so that those for plane strain can simply be deduced from them by setting $n = 0$ and $T = S \sin \phi + c \cos \phi$.

It should also be mentioned that the variation in soil strength parameters, $c$ and $\phi$, has also been considered in these equations. Variation in $c$ and $\phi$ can be due to the nonhomogeneity of the soil mass. It can also be due to the difference in shear strain at different points. This later relation has already been used in obtaining the load–deflection behavior of structures in contact with soil. Therefore, the ZEL method can be reasonably used for consideration of the stress level effect on soil shear strength properties and consequently foundation behavior.
The main objective of this work is to estimate the bearing capacity and predict the load–displacement behavior of foundations, considering the stress level dependency of soil shear strength. It is done by incorporating the original ZEL equations with the capability of considering the variations of soil cohesion and friction angle. This ability is employed in a computer code in which variations of soil shear strength are related to variations of stress level in the soil mass.

In the following parts, the stress level dependency of soil shear strength is first investigated. Then the stress level dependent values of the bearing capacity of shallow foundations is computed and compared with existing experimental data. In computation of the bearing capacity, an associated flow rule has been assumed that makes it possible to compare the results of this method with those obtained by the traditional bearing capacity equation and suggested bearing capacity factors. As a practical advantage, some design charts have been developed to compute the bearing capacity factor, \( N_c \), for shallow foundations as a function of foundation size. In the last section, the load–displacement behavior of shallow foundations has been studied in which a non-associated flow rule is assumed.

**2. Stress level dependency of soil shear strength**

Variations of maximum friction angle obtained in standard laboratory shear tests with normal or confining pressure have been widely observed. It has been recognized that the peak friction angle of soils decreases with an increase in stress level, and the Mohr failure envelope is a curve rather than a straight line [9,27–29]. However, there is much evidence showing that the critical state friction angle is constant, since a soil sample reaches a constant strength at the critical state [30,31].

There are relationships between normal or confining pressure and the soil angle of dilation from laboratory tests. Bolton [29] proposed to correlate the maximum friction angle to the soil relative density, \( D_r \), and applied effective stress, \( \sigma \), as the following simplified forms:

\[
\phi_{\text{max}} = \phi_{c,s} + 5 k (\text{plane-strain condition}), \quad (5a)
\]

\[
\phi_{\text{max}} = \phi_{c,s} + 3 k (\text{axi-symmetric condition}), \quad (5b)
\]

\[
I_k = D_r (\ln Q - \ln R). \quad (5c)
\]

In these equations, \( \phi_{\text{max}} \) is maximum mobilized friction angle, \( \phi_{c,s} \) is critical state friction angle, \( v_{\text{max}} \) is maximum dilation angle, \( I_k \) is dilatancy index, \( D_r \) is soil relative density (in decimal), \( \sigma \) is the effective stress (in kPa) and \( Q \) and \( R \) are constants. Bolton [29] recommended to use \( Q = 10 \) and \( R = 1 \). Kumar et al. [32] performed a number of shear tests on Bangalore sand and utilized these equations to express the stress level dependency of test specimens.

Clark [10] performed a series of triaxial tests on a dense silica sand with density index of 88%, mean grain size \( (d_{50}) \) equal to 0.2 mm, coefficient of uniformity, \( C_u = 1.69 \), and density of 15 kN/m³ at different confining pressures [10]. Based on these results, the following simple equation was suggested to correlate the maximum friction angle to the level of confining pressure [10]:

\[
\phi = A (\sigma)^M. \quad (6)
\]

In this equation, \( \phi \) is the maximum mobilized friction angle as a function of \( \sigma \), \( \sigma \) is confining pressure \( (\sigma) \) in the triaxial test or normal stress, \( \sigma_n \), in the direct shear test, \( A \) is a factor that can be considered as the peak friction angle at unit confining pressure and \( M \) is an exponent. Using this equation, there is no need to determine the relative density in the laboratory, and the coefficients can be determined simply by standard shear tests on soil samples.

These coefficients were determined by Clark [10] for the soil used in his study from triaxial test results at different confining pressures.

The direct shear test results of Kumar et al. [32] have been utilized to check the validity of this relatively simple equation for other soil types in a direct shear test. Kumar and his co-workers have performed extensive laboratory shear tests on Bangalore sand. This type of sand consists of an average grain size \( (D_{50}) \) of 0.62 mm, \( D_{10} = 0.23 \) mm, \( D_{30} = 0.40 \) mm and \( D_{90} = 0.78 \) mm. Also \( C_c = 3.4 \) and \( C_s = 0.9 \). Therefore, this sand can be classified as poorly graded sand (SP). Maximum and minimum densities were found to be 18.13 kN/m³ and 14.3 kN/m³, respectively. Direct shear tests have been performed at densities of 15.21 kN/m³, 16.19 kN/m³, 17.17 kN/m³ and 17.66 kN/m³, corresponding to relative densities equal to 28.5%, 55.6%, 79.6% and 90.6%, respectively. Variations of soil friction angle versus normal effective stress on a semi log scale plot show the validity of the suggested equation by Clark, for this type of sand [33].

**3. Numerical solution procedure**

**3.1. Stress field**

The ZEL equations can be solved by numerical techniques. To do this, according to Jahanandish [24], starting from a boundary on which all necessary information, i.e. values of \( r, z, S \) and \( \psi \), are readily known, the equations are written in finite difference form and the unknowns are determined at the next points of the domain. For example, considering Figure 2, assume the boundary \( A_1A_3 \) to be a boundary with predefined values of \( r, z, S \) and \( \psi \). The ZEL equations can be numerically solved to find unknowns in point \( B_1 \), based on \( A_1 \) and \( A_2 \). A similar procedure can be used for points \( B_2 \) and \( C_1 \). For details of the calculations and assumptions, one may refer to [24].

**3.2. Strain and displacement field**

The ZEL method can also be used to find the velocity field. Since the Zero Extension Lines are lines of zero axial strains, the ZEL would be used as rigid links that can move or rotate without axial deformation. As a consequence, for a given deformation boundary condition, the generated displacements in the ZEL net can be computed by the following equation:

\[
\frac{d u}{d w} = -\frac{dz}{dr}. \quad (7)
\]
In this equation, $u$ and $w$ are horizontal and vertical displacements. The finite difference forms of these equations can be used to find the deformed ZEL net for further computations which are presented in the Appendix. When the velocity field has been found, the maximum shear strains can be determined. The developed shear strains can be determined from the displacement field obtained from the boundary deflection. The shear strains can then be used to find the mobilized friction angle at different points of the ZEL net by using the relationship between $\gamma_{\max}$ and $\sin \phi_{mob}$.

3.3. Numerical solution and the developed code

Simultaneous solution of the ZEL equations requires a system of four equations and four unknowns, which as stated before, is done using a triple point strategy. A computer code in MATLAB was developed to solve the ZEL equations, considering the stress level dependency of soil friction angles. This is done by a definition of the geometry of the problem, insertion of soil parameters and properties, initiation of stress field, and computation of load–displacement behavior by the ZEL method. Initiation of the stresses is performed by the Jaky formula (presented in many soil mechanics texts, such as [34]) as follows:

$$k_0 = 1 - \sin \phi_{c,s}$$

This code is capable of solving the ZEL equations for both axi-symmetric and plane strain problems and is comprised of three different computational blocks and some supplementary functions. Major parts of the code can be found in the Appendix of this paper, along with a flowchart showing the computational procedure illustrated in Figure A.1.

3.4. Boundary condition

It is necessary to consider an appropriate boundary condition in particular for a rough base foundation. In this research, the boundary condition of Bolton and Lau is assumed [5]. They supposed that a rigid triangular wedge (or cone in axi-symmetric problems) is formed beneath the foundation, as shown in Figure 3. According to Meyerhof, it is inclined at angle $\alpha$ equal to $\pi/4 + \phi/2$ [2,5].

4. Bearing capacity problem

As stated before, the first objective of this work is to study the effect of the stress level on the bearing capacity of shallow foundations. First, the developed computed code is verified by traditional methods and, then, the code is employed to estimate the bearing capacity of shallow foundations, considering the stress level influence on soil shear strength parameters.

4.1. Verification of the developed computer code

The ZEL net and foundation pressure distribution for the case of a weightless soil with $\phi = 40^\circ$ in which the second bearing capacity factor, $N_q$ can be computed, is shown in Figure 4. Table 1 shows the computed values of $N_q$ in comparison with closed form solutions for strip foundations. The values are reasonably the same as the closed form solutions found in the literature.

As stated earlier, computation of the third bearing capacity factor, $N_r$, has been performed by many authors. Among them, several authors have applied the slip line method of Sokolovskii [35] to compute this factor for different cases of smooth and rough base foundations in plane strain and axi-symmetric problems [36,37]. Since the assumptions made by Bolton and Lau have the most similarity with the method implemented in this study, and their boundary conditions have been considered in the analyses, the results obtained by them have been compared with the results of this paper [5]. The third bearing capacity factor, $N_r$, was also computed for a range of soil friction angles for both strip and circular foundations. The surcharge pressure was kept so low that the value of the dimensionless factor, $\Omega$, is not more than 0.05; this prevents any superposition assumption. This factor was computed under both plane strain and axi-symmetric conditions for smooth and rough base foundations. Figure 5 shows the ZEL net for a rough base circular foundation of 2.0 m in width on a frictional heavy soil with $\phi = 40^\circ$ and $\gamma = 20$ kN/m$^3$. The stress distribution beneath the foundation is also shown in this figure.

Table 2 shows the computed values in comparison with the results of Bolton and Lau [5], which have been calculated by the method of stress characteristics. The computed values show reasonable agreement with the results obtained by Bolton and Lau [5] based on the method of stress characteristics. The slight difference between the results can be related to the assumptions made in this study, the coarseness of the mesh and the assumed value of $\Omega$.

4.2. Bearing capacity considering stress level

It was stated that foundation size has an important effect on the bearing capacity of shallow foundations. The previous methods neglect this effect and present similar values for the bearing capacity factor, $N_r$, for different size foundations, by assuming a constant friction angle for the entire soil body. These values are valid only for small foundations. For large foundations, however, these are questionable and seem to be over-estimated. For example, Eslami and Gholami inspected some case studies and suggested using in situ tests to eliminate difficulties in computing the bearing capacity coefficients [38]. There are also some modifications on the bearing capacity equation to prevent unlimited increase in bearing capacity by increasing the foundation size. Bowles [39] suggested a reduction factor for the third bearing capacity term, as follows:

$$r_r = 1 - 0.25 \log \left( \frac{B}{2} \right).$$

Table 1: Values of $N_q$ for strip foundations by ZEL and closed form solution.

<table>
<thead>
<tr>
<th>$\phi$ (°)</th>
<th>$N_q$ (closed-form)</th>
<th>$N_q$ (ZEL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>20</td>
<td>6.4</td>
<td>6.3</td>
</tr>
<tr>
<td>30</td>
<td>18.4</td>
<td>18.1</td>
</tr>
<tr>
<td>40</td>
<td>64.1</td>
<td>62.5</td>
</tr>
</tbody>
</table>

Figure 3: Boundary condition assumed by Bolton and Lau [5] for rough base foundations.
Table 2: Values of $N_γ$ for circular rough foundation.

<table>
<thead>
<tr>
<th>$\phi$ (°)</th>
<th>Strip foundations</th>
<th>Circular foundations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rough base</td>
<td>Smooth base</td>
</tr>
<tr>
<td>10</td>
<td>1.71</td>
<td>1.7</td>
</tr>
<tr>
<td>20</td>
<td>5.97</td>
<td>6.1</td>
</tr>
<tr>
<td>30</td>
<td>23.6</td>
<td>23</td>
</tr>
<tr>
<td>40</td>
<td>121</td>
<td>120</td>
</tr>
</tbody>
</table>

In this equation, $B$ is the foundation size in meters and $r_γ$ is the reduction factor for the third bearing capacity term. The advantages and limitation of this reduction factor will be checked along with more precise analyses with the aid of the stress level based ZEL method.

Meyerhof [27] and De Beer [28] suggested using the value of the friction angle corresponding to the mean normal effective stress, $\sigma_m$, along the failure surface obtained by the following relationship:

$$\sigma_m = 0.25q_{ult}(1 - \sin \phi). \quad (10)$$

Using the relationships between stress states in a Mohr circle of stresses, the value of confining pressure, $\sigma_3$, along the failure surface can be determined. The corresponding value of the soil friction angle can then be found as a function of ultimate bearing capacity. An iterative procedure is required for convergence.

At this point, the advantage of the ZEL method is employed to predict the actual bearing capacity of shallow foundations, with consideration of the stress level dependent nature of subsoil elements. To do this, a practical case is investigated. It should be noted that since the large scale foundations are beyond the capacities of common testing equipment, the centrifuge tests have been developed for modeling large foundations in the laboratory. In 1985, Kimura and his coworkers performed a series of centrifuge tests on compacted layers of Toyoura sand in a dense state to explain the scale effect of shallow foundations on the bearing capacity [40]. Toyoura sand has been used in many experimental programs, and its properties have been reported by several authors [15,41,42]. In this work, a summary of Toyoura sand properties, which are assumed in the analyses, is presented in Table 3.

To take the effect of stress level into account, it is necessary to define a relationship between the soil maximum friction angle and confining pressure. The stress level dependency of the soil friction angle has been expressed by the Bolton suggested relationship [29], which was presented earlier. It relates the state of dense sand (its relative density) to its maximum mobilized friction angle as a function of stress level in different laboratory shear tests.
By incorporating this relationship, a number of analyses were performed to calculate the bearing capacity factor, $N_γ$, for different size foundations tested by Kimura et al. [40] in a centrifuge facility. Bearing capacity tests were performed for model rough footings with three different breadths (B) of 20, 30 and 40 mm at different embedment depths. The experiments were conducted both in the centrifuge with accelerations of log, 20g and 40g, and in the laboratory, i.e. at l0g, resulting in equivalent foundations of different sizes. The rough base boundary condition of Bolton and Lau [5] was utilized for the analyses. Figures 6 and 7 show the results of the analyses for two different size circular foundations, i.e. 0.3 m and 1.6 m, by the ZEL method, along with foundation pressures at failure. In these figures, the variation of soil maximum friction angle is also depicted, showing a decreasing tendency with an increase in stress level; e.g. the lowermost values near the foundation base. Comparison between these two cases shows that the soil friction angle is generally higher for smaller foundations, resulting in higher bearing capacity factors, $N_γ$. At this point, it is necessary to state that although Toyourasand is highly non-associative, a comparison between the results obtained by traditional methods, without considering the stress level dependency of the soil friction angle, and the results obtained by the current study, in which such a dependency is considered, an associative flow rule is assumed. Therefore, the value of $N_γ$ can be compared to those suggested in the literature and shows the influence of the stress level dependency of the soil friction angle.

Table 4 shows the obtained data from calculations and experimental tests. For each size, three tests have been carried out by Kimura and his coworkers. For this work, the average values are presented in the table with approximately 10% error. In this table, another comparison was made with the suggested formula of Meyerhof [2] for critical state and peak friction angles. Values corresponding to peak friction angle have been calculated for two limits of the suggested range for $\phi_{peak}$, i.e. 44° and 50°, resulting in $N_γ$ equal to 211 and 873, respectively.

The results show that the ZEL method can reasonably provide the bearing capacity factor, $N_γ$, for shallow foundations, considering the stress level dependency of the soil friction angle. It is also remarkable that while the minimum value for the peak friction angle (in the Meyerhof equation) provides almost the lowermost value of $N_γ$, its maximum value is well above the uppermost value of $N_γ$ observed in experimental tests. It should also be noted that the value of $N_γ$ corresponding to the critical state soil friction angle is reasonably below the lowermost values obtained from test results and hence is highly conservative. It is worth mentioning that the lowermost value obtained from the ZEL method is 53 for the critical state friction angle, and the uppermost value is ranged between 246 and 956 for the peak friction angle.

By performing a number of analyses and employing the relationships for soil friction angle variations at different stress levels, a number of analyses were performed to calculate the bearing capacity factor, $N_γ$, for different size foundations tested by Kimura et al. [40] in a centrifuge facility. Bearing capacity tests were performed for model rough footings with three different breadths (B) of 20, 30 and 40 mm at different embedment depths. The experiments were conducted both in the centrifuge with accelerations of log, 20g and 40g, and in the laboratory, i.e. at l0g, resulting in equivalent foundations of different sizes. The rough base boundary condition of Bolton and Lau [5] was utilized for the analyses. Figures 6 and 7 show the results of the analyses for two different size circular foundations, i.e. 0.3 m and 1.6 m, by the ZEL method, along with foundation pressures at failure. In these figures, the variation of soil maximum friction angle is also depicted, showing a decreasing tendency with an increase in stress level; e.g. the lowermost values near the foundation base. Comparison between these two cases shows that the soil friction angle is generally higher for smaller foundations, resulting in higher bearing capacity factors, $N_γ$. At this point, it is necessary to state that although Toyourasand is highly non-associative, a comparison between the results obtained by traditional methods, without considering the stress level dependency of the soil friction angle, and the results obtained by the current study, in which such a dependency is considered, an associative flow rule is assumed. Therefore, the value of $N_γ$ can be compared to those suggested in the literature and shows the influence of the stress level dependency of the soil friction angle.

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By performing a number of analyses and employing the relationships for soil friction angle variations at different stress levels,
Table 4: Bearing capacity factor, $N_γ$, obtained from experimental tests and theoretical methods.

<table>
<thead>
<tr>
<th>$B$ (m)</th>
<th>$N_γ$ (Experiment)</th>
<th>$N_γ$ (ZEL)</th>
<th>$N_γ$ (Meyerhof [2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>580</td>
<td>580</td>
<td>33°</td>
</tr>
<tr>
<td>0.3</td>
<td>450</td>
<td>510</td>
<td>33</td>
</tr>
<tr>
<td>0.6</td>
<td>350</td>
<td>410</td>
<td>33</td>
</tr>
<tr>
<td>0.8</td>
<td>300</td>
<td>370</td>
<td>33</td>
</tr>
<tr>
<td>1.2</td>
<td>270</td>
<td>320</td>
<td>33</td>
</tr>
<tr>
<td>1.6</td>
<td>250</td>
<td>300</td>
<td>33</td>
</tr>
</tbody>
</table>

* By suggested equation based on the ZEL method.
** Corresponding value obtained from the ZEL method is 53 for $ϕ_{c.s.}$ and 246–956 for $ϕ_{peak}$ (suggested equation).

levels, some design charts have been suggested to relate the actual bearing capacity factor, $N_γ$, to the foundation size by the ZEL method. In this paper, a number of these charts have been represented in Figures 8–11.

5. Load–displacement behavior

As stated before, strain and velocity fields can be found by the ZEL for any arbitrary displacement boundary condition, i.e., foundation translation or rotation. There are several problems involved in testing large scale foundations. Many foundation load tests have been conducted on small scale model footings. Therefore, numerical techniques can be a good alternative to predicting the load–displacement behavior of foundations, instead of costly and difficult foundation load tests. In this part, the ZEL method has been employed for prediction of the load–displacement behavior of shallow foundations by investigation of an existing case study.

5.1. Experimental tests of clark

An experimental program was performed by Clark to study the effect of foundation size on the bearing capacity and load–displacement behavior of foundations on strong soils [10]. Small to large scale circular foundations were tested in the program. As stated by Clark, a model test footing (43.7 mm in diameter) was tested in a centrifuge apparatus at accelerations of 1, 10, 40, 100 and 160g. The resulting diameters of the prototype foundations were 0.044 m, 0.437 m, 1.75 m, 4.37 m and 6.99 m, respectively. A dry dense sand was used for tests with $γ_d = 15.04 \text{kN/m}^3$ [10]. The peak friction angle was determined to be ranged between $39°$ and $49°$. An average critical state soil friction angle of $36°$ was assumed for the analyses regarding triaxial test results. Complete test data on this soil was described by Clark and, for the purpose of this work, the relationship between $sinϕ_{mob}$ and maximum shear strain, $γ_{max}$, was developed, according to laboratory test results. As stated in the literature, the ZEL method requires the relationship between $sinϕ_{mob}$ and $γ_{max}$ to compute the complete load–displacement field [17,22–24]; this is shown in Figure 12. The soil maximum dilation angle was estimated to vary between $5°$ and $20°$, considering triaxial test results. To prevent difficulties in numerical solution, a dilation angle equal to $16°$ was assumed for all analyzed cases. Figure 13 shows the ZEL net and velocity field for a 0.44 m diameter foundation. The results show good agreement.
The results show that for small foundations, there is an apparent peak bearing pressure, which is gradually disappeared by increasing the foundation size. Therefore, smaller foundations over a certain soil type show a general shear failure mechanism, whereas this mode of failure is getting more localized for larger ones over the same soil. This transition in foundation behavior can be captured by the stress level based ZEL method described in this paper. This phenomenon suggests that for relatively large foundations, lower values of soil friction angle are mobilized, and it tends to a transition between the modes of failure. When the major part of the soil undergoes very high levels of stress, maximum mobilized friction angles cannot exceed the critical state value and, as a consequence, a local shear failure without a peak pressure could be observed.

Another important conclusion is that the load–displacement curve obtained from the ZEL method can reasonably be used for prediction of the ultimate bearing capacity of foundations. In spite of the bearing capacity obtained from the traditional equation, in the load–displacement approach, it is possible to answer the raised questions on the magnitude of the relative settlement at failure, possible failure mechanism and the peak bearing pressure. This provides a relatively thorough insight into foundation behavior under the increasing trend of loading, and reveals whether a peak bearing pressure could be expected or that the foundation will continue to settle without reaching or approaching an apparent ultimate pressure. Therefore, by a stress level ZEL approach, it is possible to distinguish foundation behavior over different soil types regarding foundation sizes, while a small foundation on a relatively dense soil may show a general shear failure mechanism, a larger foundation on the same soil would behave in a different way. As a result, the distinction between different foundation behaviors is not related to soil conditions alone; the foundation size effect through a stress level approach should also be taken into account.

6. Conclusions

The ZEL method, which has been developed for many problems in soil mechanics, was employed in this paper for investigation of shallow foundation behavior, considering the stress level effect. Since the variations of soil shear strength parameters are included in the ZEL equations, it is possible to use these equations to consider the stress level dependency of the soil friction angle. This property of the ZEL method...
was utilized through a developed numerical computer code, and the bearing capacity and load–displacement behavior of foundations have been studied using comparisons with existing experimental data.

The results show that when the bearing capacity factor, \( N_{\gamma} \), has been experimentally found to be decreasing with an increase in foundation size, this phenomenon can be reasonably captured in the stress level dependent ZEL method. Comparison between the results obtained from the ZEL method and those obtained experimentally revealed that while the stress level effect is considered, the ZEL method can provide reasonable predictions on both the bearing capacity and load–displacement behaviors of shallow foundations. The bearing capacity from a load–displacement approach, taking the stress level effect into account, makes it possible to consider the displacements besides foundation pressure. Further study on the results showed the transition between general shear failure and local shear failure modes when the foundation size increases. This phenomenon can be considered as a direct result of the stress level dependency of soil shear strength parameters, which are predicted by the stress level based ZEL method and the developed computer code. Such observations suggest using a load–displacement curve to find the actual ultimate load on the foundations, rather than a relatively rough estimate from the conventional bearing capacity formula. The load–displacement approach, based on the presented ZEL method, by considering the stress level effect, prevents two common problems involved in the bearing capacity equations: first, taking a constant value of the soil friction angle without the foundation size effect, widely observed by the researchers through experimental tests, and second, no consideration of the foundation relative settlement at failure, which is important for the performance of the structure. The stress level based method presented in this paper can also capture the possible mechanism of failure under different size foundations over the same soil type.

Appendix

Finite difference forms of the equations

The ZEL method provides the plasticity equations along the Zero Extension Lines (lines along which, axial strains are zero). The main differences between this method and the method of stress characteristics are that the ZEL method can be applied to both associative and non-associative flow rule conditions, and the ZEL method can be used to find the strain field, as well as the stress field; hence it can be used for load–displacement problems. Once the strain field obtained by a known displacement at the displacement boundary has been found, and if the relationship between the mobilized soil friction angle, \( \sin \phi_{mob} \), and the maximum shear strain, \( \gamma_{max} \), exist, a complete load–displacement behavior of any structure in contact with soil can be predicted; this is shown in the literature [17,20–24].

The plasticity equations along the ZEL directions have been derived by Anvar and Ghahramani [23] and Jahanandish [24], and more details on the derivation of these equations can be found in their work. There are four equations and four unknowns at each point, e.g. for an arbitrary point like C, calculations should be performed to find the unknowns at point C from the existing data of the previous two points, namely, A and B. For terms without a subscript index, the averaged values between two successive points should be used. For example, angle \( \psi \) is initially set equal to \( \psi_A \), and, after the first round of iterations, it is replaced by the averaged value of \( \psi_A \) and \( \psi_C \) along the positive direction. The finite difference forms of the equations are as follows:

\[
\begin{align*}
\text{For + tive ZEL:} & \quad \frac{(z_C - z_A)}{(r_C - r_A)} = \tan(\psi + \xi) \\
\text{For - tive ZEL:} & \quad \frac{(z_C - z_B)}{(r_C - r_B)} = \tan(\psi - \xi).
\end{align*}
\] (A.1)
Jahanandish equations [24]:

Along the plus (+) ZEL:
\[
\begin{align*}
& (S_C - S_A) + \frac{(T_C - T_B)}{\Delta_{BC}} - \Delta_{AC} + \frac{2T}{\cos \nu} \left( (\psi_C - \psi_A) ight. \\
& - \sin \nu \left( \frac{\psi_C - \psi_B}{\Delta_{BC}} \right) \Delta_{AC} \\
& + f_z \sin (\psi + \xi) \right) \Delta_{AC}
\end{align*}
\]

Along the minus (−) ZEL:
\[
\begin{align*}
& (S_C - S_B) + \frac{(T_C - T_A)}{\Delta_{BC}} - \Delta_{AC} - \frac{2T}{\cos \nu} \left( (\psi_C - \psi_B) ight. \\
& - \sin \nu \left( \frac{\psi_C - \psi_A}{\Delta_{BC}} \right) \Delta_{AC} \\
& + f_z \sin (\psi - \xi) \right) \Delta_{BC}
\end{align*}
\]  

\[ \Delta_{AC} = \sqrt{(r_A - r_C)^2 + (z_A - z_C)^2} = \delta^+ \]  
\[ \Delta_{BC} = \sqrt{(r_B - r_C)^2 + (z_B - z_C)^2} = \delta^- . \]  

Displacement equations:

The displacement field can be found by the following equations:

\[
\begin{align*}
& dudr + dvdz = 0 \\
& \Rightarrow \begin{cases}
(\hat{u}_{B2} - \hat{u}_{A2})(r_{B2} - r_{A2}) + (\hat{w}_{B2} - \hat{w}_{A2})(z_{B2} - z_{A2}) = 0 \\
(\hat{u}_{B2} - \hat{u}_{A3})(r_{B2} - r_{A3}) + (\hat{w}_{B2} - \hat{w}_{A3})(z_{B2} - z_{A3}) = 0.
\end{cases}
\]  

(A.4)
found. It can be related to the mobilized soil angle of dilation. Maximum shear strain can be calculated as follows:

\[
\gamma_{\text{eff}} = \left[ \cos(\psi + \xi) \frac{\partial u}{\partial \xi} - \cos(\psi - \xi) \frac{\partial u}{\partial \eta} + \sin(\psi + \xi) \frac{\partial w}{\partial \xi} + \sin(\psi - \xi) \frac{\partial w}{\partial \eta} \right] \cos \nu. \tag{A.5}
\]

**Description of the developed computer code and computational procedure**

**Block 1: Input data.** In the first block, the input parameters are defined, containing soil geotechnical properties necessary for the equations and geometry of the problem. These parameters include: soil density, shear strength parameters, relative density, foundation size (width) and failure mechanism (smooth or rough base foundation), surcharge pressure etc. There are also some controlling parameters, such as number of iterations and the convergence criterion (precision of the result).

**Block 2: Calculations.** In this block, the calculation procedure starts from a boundary on which all necessary data are prescribed. For the certain case of the bearing capacity problem, this boundary is the ground surface over which a surcharge pressure may or may not exist. It is, however, necessary to define a very small value for the surcharge pressure in the absence of any actual surcharge load. It is necessary to avoid a trivial solution for the partial differential equations of the ZEL method. A triple-point procedure is used to calculate the unknown variables at the third point using the finite difference forms of the ZEL equations.

There is a subroutine that performs the numerical solutions for every point within the domain. Once the initial stresses are computed for an arbitrary point in the domain, an iteration procedure is performed to compute the corresponding mobilized friction angle, in the case of a stress level dependent soil friction angle, as a function of stress level, i.e., confining pressure, \( \sigma_s \). In this procedure, the stresses are first computed and then corresponding mobilized friction angles are obtained. Since the stresses depend on values of the soil mobilized friction angle, the procedure is repeated until the convergence criterion is met (not significant change in the calculated values from previous step). There is also another function within which the mobilized soil friction angle is computed. In this function, the dependency of the soil friction angle to the stress level is defined. Later, the computation for the next point in the domain is performed to compute all unknown variables in the field.

Computation of the stress field is also possible by applying a displacement boundary. This procedure can be used to predict load–displacement behavior. To do this, it is necessary first to construct the ZEL field. This can be done by initializing the stress field. Since the ZEL net is assumed to be constant during the analysis, it is required to find an appropriate ZEL field. According to James and Bransby, for a major part of shearing, the soil angle of dilation remains constant and hence this fact can be used as a basis for initializing the ZEL field [18]. Therefore, at the beginning, the ZEL field is constructed for an initial value of the soil friction angle and the soil angle of dilation. Regarding the variations of soil friction angle at each point during the analysis, and the stress level dependency of this parameter, and considering that for higher shear strains, a critical state soil friction angle is mobilized, it seems to be reasonable to initialize the ZEL net by \( \phi \) and \( \nu_\text{max} \) at each point. It also prevents difficulties involved in highly non-associated fields because of the coaxiality assumption, which forces the direction of the principal strains and stresses to be the same. The developed code is capable of taking a variable \( \nu \) field into account through a supplementary function. To do this, an iteration procedure is performed after the first initialization of the field over the value of \( \nu_\text{max} \) to find corresponding values, depending on stress levels at each point, regarding the variations of the soil angle of dilation with the stress level. There is a supplementary function in the code to compute variations of soil dilation angle. A constant \( \phi \) and constant \( \nu \) field can also be obtained for some certain cases, for example, an associative flow rule assumption in which \( \nu = \phi \), with constant values of soil friction angle and dilation angle in the entire domain. Such a procedure can be used for computation of the ordinary bearing capacity factors.

Now the ZEL net is ready and can be used for computation of the bearing capacity or prediction of the load–displacement behavior. If the bearing capacity is required, it is sufficient to introduce the dependency of the soil maximum mobilized friction angle to the stress level and proceed with finding the appropriate stress field.

If a load–displacement analysis is required, another computation should be performed. It is remarkable that the stress field obtained from the initialization is not the actual initial stress field, since it corresponds to a limit state at which higher soil friction angles are mobilized. However, at the beginning of foundation loading, the subsoil is in at-rest condition. Therefore, another procedure is necessary by which initial stresses are computed by a k_0 procedure. In this part, the initial stresses in depth are computed, regarding the value of the soil coefficient of the earth pressure at rest. This is done according to the Jaky formula.

Then the current stress field is used for the first step of loading and displacements. It should be noted that the smaller the applied displacements, the higher the accuracy of the solution, considering the stress level dependency of the soil stress–strain curves. Therefore, applying any displacement increment results in a velocity and strain field by which the maximum shear strains can be calculated at each point. This shear strain along with the stress level of the previous step is used to find the mobilized friction angle of the next step and accordingly the stress field. This can be done by using the relationship between \( \gamma_{\text{max}} \) and \( \sin \phi_{\text{mod}} \), which are stress level dependent functions obtained from laboratory tests at different confining pressures.

**Block 3: Output.** In this block, output data is presented. Output contains all unknowns within the field, i.e., mean stress, angle \( \psi \), and \( x \) and \( z \) coordinates of any point in the domain, mobilized friction angle of corresponding points, ultimate bearing pressure and the bearing capacity of the foundation or the load–displacement curve.

The flowchart of the developed computer code is shown in Figure A.1.

**References**

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