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APPLICATION OF THE ZEL METHOD IN THE PREDICTION OF FOUNDATION BEARING CAPACITY CONSIDERING THE STRESS LEVEL EFFECT

M. Veiskarami, M. Jahanandish and A. Ghahramani
Shiraz University, Shiraz, Iran

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There are a number of factors affecting the bearing capacity of shallow foundations. Among them, the scale effect can be mentioned as one of the most important factors. Unlike the theoretical equations, experiments show that the bearing capacity of foundations does not increase without limit when the foundation size increases. The effect of stress level on soil shear strength parameters has been known as the main reason for this observation. The method of the zero extension lines (ZEL) for the solution of plasticity problems in soil mechanics has been utilized to take this effect into account by incorporating the stress level – dependent soil friction angle. The bearing capacity of shallow foundations is then computed with the aid of this method, showing a decreasing tendency in the third factor, N_γ, which is the main contributor in shallow foundations. Comparisons have been made with experimental data, showing good consistency between experiments and theoretical predictions with the ZEL method.

Introduction

Among many different approaches to determine of the bearing capacity of shallow foundations, the well-known formula of Karl Terzaghi (1943) has been widely accepted for the bearing capacity as follow [1]:

$$q_{ult} = cN_c + qN_q + 0.5\gamma BN_\gamma. \tag{1}$$

In this equation, q_{ult} is the ultimate bearing capacity, c is the soil cohesion, q is the surcharge pressure, B is the foundation width, γ is the soil density, and the N_i coefficients are the bearing capacity factors as functions of soil friction angle. The third term has been known as the main contributor in the bearing capacity of shallow foundations, however, unlike the first two factors, i.e., N_c and N_q, the third factor is the most challengeable one. There are several values for the third factor suggested by different authors [1-7]. Although all of these methods are generally based upon a limit analysis solution, there are differences between their assumptions on the boundary conditions and taking the effect of soil weight into account. It is also remarkable that the bearing capacity of shallow foundations is computed based on an associative flow rule assumption. There are a few studies on the nonassociative flow rule effect on the bearing capacity. Michalowski (1997) showed that, in general, such assumption results in a lower bearing capacity factor, N_γ [6].

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The third term suggests an increasing tendency in the bearing capacity with increase in foundation size. However, data from De Beer (1965), Bolton and Lau (1989), and Clark (1998) show that the bearing capacity of shallow foundations does not increase with size without limit [8-10]. Recently, other investigations showed similar results (Cerato, 2005; Cerato and Lutenegger, 2006; Kumar and Khatri, 2008; Yamamoto et al., 2009) [11-14].

On the other hand, Fellenius and Altaee (1994) showed that for settlements even above 10% of the footing width (or diameter), the foundation behavior is not that of approaching or reaching an ultimate failure mode [15].

These effects are called *scale effects* which are related to the stress level experienced by the soil. Investigations showed that the soil friction angle is generally a function of the stress level, resulting in a curved Mohr-Coulomb failure envelope (Bolton, 1986; Clark, 1998; Kumar et al., 2007) [10, 16, 17].

The ZEL method makes it possible to find both stress and strain fields if the displacement boundary, i.e., the foundation settlement, is known. Therefore, one of the advantages of derivation of plasticity equations along the ZEL directions is that the solution of deformation problems is also possible, while the stress field alone can also be found from a known stress boundary. In this study, the theory of the ZEL is utilized to take the stress level effect into account in the computation of the bearing capacity of shallow foundations. A computer code in MATLAB was developed to solve numerically the ZEL equations with stress level dependence considerations. An existing experimental test was also studied.

The Zero Extension Lines Method: Theory and Applications

Bearing capacity and load-displacement behavior of foundations can be investigated with the aid of plasticity problems in soil mechanics. The slip line method of Sokolovski (1960) has been known as one of the most effective methods in solving plasticity problems [18]. The method has been developed in recent years for many problems in foundation engineering. For example, extensions for circular and strip foundations (Bolton and Lau, 1993), for ring footings (Kumar and Ghosh, 2005), for rough strip foundations (Kumar and Kouzer, 2007), for foundations on unsaturated soils (Veiskarami et al., 2009), and other cases can be cited [5, 19-21]

The idea of using the ZEL for obtaining the strains and deformations in the soil mass, and thereby predicting the load-deflection behavior of structures in contact with soil, was introduced by Roscoe (1970) [22]. James and Bransby (1971) used them for strains and deformations prediction behind a model retaining wall [23]. Habibagahi and Ghahramani (1979) and Ghahramani and Clemence (1980) calculated soil pressures by considering the force-equilibrium of soil elements between the zero extension lines [24, 25]. Jahanandish (1988) and Jahanandish et. al. (1989) presented the methodology for finding the load-deflection behavior of the foundation and retaining walls on the basis of the ZEL theory [26, 27]. The general forms of these lines were considered in this methodology and the variations of soil shear strength parameters, i.e., cohesion and friction angle, with the induced shear strain due to the deflection of the structure in contact with soil, were also taken into account. Then, Anvar and Ghahramani (1997) used the matrix method for derivation of differential equilibrium-yield equations along the stress characteristics in the plane strain condition and transferred them along the ZEL [28]. This approach was important since it allowed integration of the equations along the zero extension directions. Later, Jahanandish (2003) considered the more general case of axial symmetry and obtained the equilibrium-yield equations along the ZEL by direct transformation, independent of the stress characteristics [29]. Details of these derivations can be found in Anvar and Ghahramani 1997) [28] and Jahanandish (2003) [29]. The final forms of these equations are presented here.

The zero extension directions are defined by

$$\frac{dz}{dx} = \tan\left(\psi \pm \xi\right), \tag{2}$$

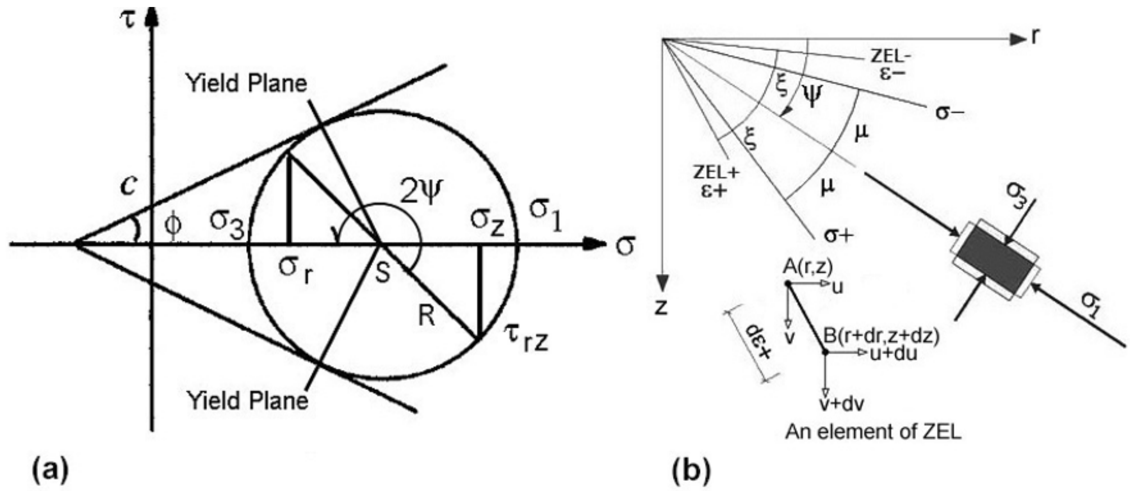

Fig. 1. Directions of yielded planes and the ZEL directions: a) Mohr circles of stress and b) stress and strain characteristic (ZEL) directions with respect to the horizontal direction.

where $\xi = \pi/4 - \nu/2$, and the minus sign (–) stands for one direction, and the plus sign (+) for the other, which are shown in Fig. 1 along with their position with respect to yielded planes which are in general different from each others. It is noticeable that if associativity is assumed, the yielded planes coincide with the ZEL directions.

Based on Anvar and Ghahramani's (1997) derivation, the equilibrium-yield equations along the ZEL for the plane strain problem are as follows [28]:

Along the – ZEL

$$dS - 2(S\tan\phi + c)\left(\bar{\alpha}d\psi + \bar{\zeta}\frac{\partial\psi}{\partial\varepsilon^{+}}d\varepsilon^{-}\right) = X\bar{\beta}(\tan\phi dz + \bar{\alpha}dx) - Z\bar{\beta}(\tan\phi dx - \bar{\alpha}dz) +$$
$$+(S - c\tan\phi)\left(\tan\phi d\phi - \frac{1}{\cos\phi}\frac{\partial\phi}{\partial\varepsilon^{+}}d\varepsilon^{-}\right) + \left(\tan\phi dc - \frac{1}{\cos\phi}\frac{\partial c}{\partial\varepsilon^{+}}d\varepsilon^{-}\right); \tag{3a}$$

Along the + ZEL

$$dS + 2(S\tan\phi + c)\left(\bar{\alpha}d\psi + \bar{\zeta}\frac{\partial\psi}{\partial\varepsilon^{-}}d\varepsilon^{+}\right) = -X\bar{\beta}(\tan\phi dz - \bar{\alpha}dx) + Z\bar{\beta}(\tan\phi dx + \bar{\alpha}dz) +$$
$$+(S - c\tan\phi)\left(\tan\phi d\phi - \frac{1}{\cos\phi}\frac{\partial\phi}{\partial\varepsilon^{-}}d\varepsilon^{+}\right) + \left(\tan\phi dc - \frac{1}{\cos\phi}\frac{\partial c}{\partial\varepsilon^{-}}d\varepsilon^{+}\right). \tag{3b}$$

In these equations, X and Z are the body and/or inertial forces along x and z directions and $d\varepsilon^{+}$ and $d\varepsilon^{-}$ are lengths of the differential elements along the ZEL directions. The values of $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\zeta}$ are given by

$$\bar{\alpha} = \frac{1 - \sin\phi\sin\nu}{\cos\phi\cos\nu};\ \bar{\beta} = \frac{\cos\nu}{\cos\phi};\ \bar{\zeta} = \frac{\sin\phi - \sin\nu}{\cos\phi\cos\nu}. \tag{4}$$

Based on Jahanandish's derivation (2003), the final form of the equilibrium-yield equations along the ZEL directions for the more general case of axial symmetry is [29]:

Along the - zero extension lines

$$dS + \frac{\partial T}{\partial \varepsilon^+} d\varepsilon^- - \frac{2T}{\cos\nu}\left(d\psi - \sin\nu \frac{\partial \psi}{\partial \varepsilon^+} d\varepsilon^- \right) = [f_x \cos(\psi - \xi) + f_z \sin(\psi - \xi)] d\varepsilon^-; \tag{5a}$$

Along the + zero extension lines

$$dS + \frac{\partial T}{\partial \varepsilon^-} d\varepsilon^+ + \frac{2T}{\cos\nu}\left(d\psi - \sin\nu \frac{\partial \psi}{\partial \varepsilon^-} d\varepsilon^+ \right) = [f_x \cos(\psi + \xi) + f_z \sin(\psi + \xi)] d\varepsilon^+. \tag{5b}$$

In these equations, T is the radius of the Mohr circle for stress, and n is an integer equal to 1 for the axisymmetric problems and 0 for the plane strain case; f_x and f_z are expressed by

$$f_x = X - \frac{nT}{x}(1 + \cos 2\psi);\ f_z = Z - \frac{nT}{x}\sin 2\psi. \tag{6}$$

As mentioned, these equations are more general, so that those for plane strain can simply be deduced from them by setting $n = 0$ and $T = S\sin\phi + \cos\phi$. Note that x is the measure of the radial distance for the axisymmetric problem.

It should also be mentioned that the variation in soil strength parameters c and ϕ has also been considered in these equations. Variation in c and ϕ can be due to nonhomogeneity of the soil mass. It can also be due to the difference in shear strain at different points. This later relation has already been used in obtaining the load-deflection behavior of structures in contact with soil.

Application of the ZEL Method to the Bearing Capacity of Shallow Foundations

As stated before, the ZEL method has been utilized in this paper for stress-level-dependent computation of the bearing capacity of shallow foundations. In this part, first, the bearing capacity factors are computed without considering the stress level dependence of the soil friction angle. Then, the effect of stress level is included, and the method is used to predict the actual bearing capacity of foundations. The computations are based on an associative flow rule assumption. A computer code in MATLAB is developed to solve the ZEL equations by numerical techniques, based on finite difference forms of the equations.

Bearing Capacity by Conventional Methods

This problem is a certain type of boundary value problem in soil plasticity in which the ultimate pressure beneath a foundation is determined. According to the bearing capacity formula, while the first two factors have been known to be determined by closed form solutions, the third factor has a wide range suggested by different authors. One reason is the superposition assumption that was made by a number of researchers. For example, Terzaghi (1943) assumed that the components of the bearing capacity equation can be safely superposed [1]. Bolton and Lau (1993) performed a study on the effect of surcharge pressure on computed N_γ and presented a combined bearing capacity factor, $N_{q\gamma}$, which approaches N_γ when the surcharge pressure approaches zero. This factor also approaches N_q by increasing the surcharge pressure to very high values (approaching infinity). The combined bearing capacity factor is as follows:

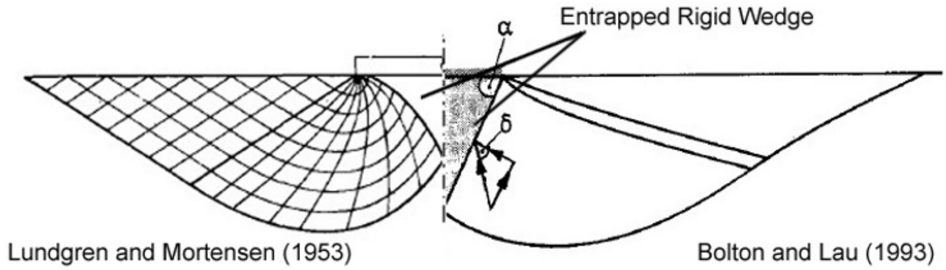

Fig. 2. Failure mechanisms for a rough base foundation assumed by Lundgren and Mortensen (1953) and Bolton and Lau (1993) (inclination of the rigid wedge, i.e., angle α is equal to $\pi/4 + \phi/2$).

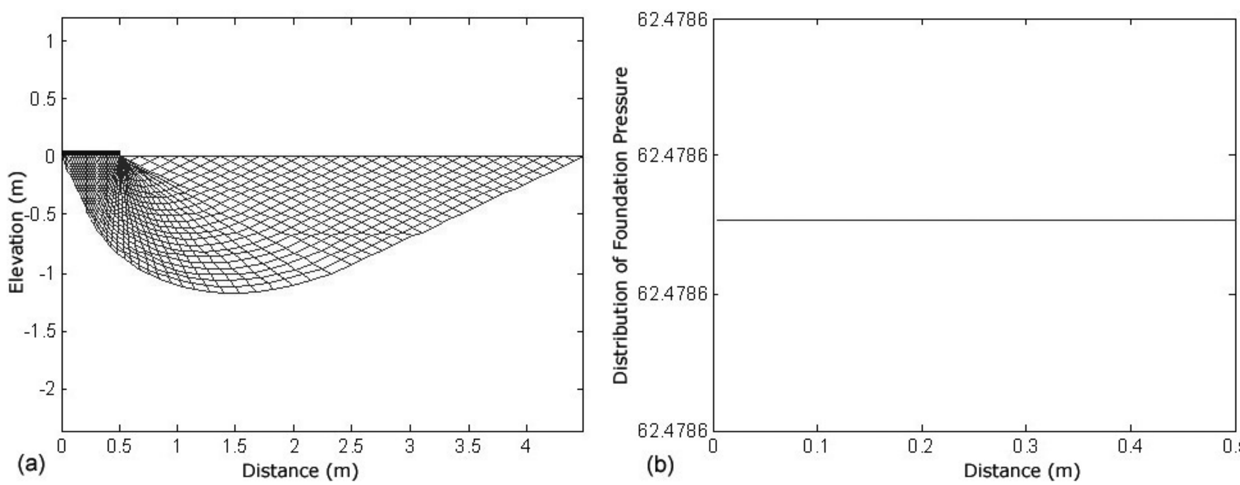

Fig. 3. Smooth base foundation over a weightless soil: a) ZEL net and b) foundation pressure distribution

$$N_{q\gamma} = \frac{q_{ult}}{0.5B\gamma + q}. \tag{7}$$

In this equation, γ is the soil density and q is the surcharge pressure.

Furthermore, differences in taking suitable boundary conditions that coincide with actual soil behavior beneath the foundation with a smooth or rough base result in different values. Figure 2 shows two typical boundary conditions for a rough base foundation suggested by Lundgren and Mortensen (1953) and Bolton and Lau (1993) [5,30]. These two mechanisms were widely used by different researchers.

With the aid of the developed computer code, the bearing capacity factors have been computed for different cases. Figure 3 shows the ZEL net and foundation pressure distribution for the case of a weightless soil with $\phi = 40^{\circ}$ in which the second bearing capacity factor, N_q, can be computed. Table 1 shows the computed values of N_q in comparison with closed form solutions for strip foundations. The results show very good similarities with closed form solutions, and better results can be obtained by further ZEL mesh refining.

Table 1. Comparison of the computed and existing bearing capacity factor N_q

ϕ, deg.	N_q (ZEL)	N_q (closed form)
0	1.0	1.0
10	2.5	2.5
20	6.3	6.4
30	18.1	18.4
40	62.5	64.1

Table 2. Values of N_γ for strip rough foundations

ϕ, deg.	ZEL	Terzaghi (1943)	Vesić (1973)	Bolton and Lau (1993)	Michalowski (1997)	Kumar and Kouzer (2007)
10	1.7	1.2	1.2	1.71	0.71	0.49
20	6	5	5.4	5.97	4.47	3.16
30	23	19.7	22.4	23.6	21.4	16.54
40	120	100.4	109.3	121	119	98.53
45	318	297.5	271.3	324	323	280.36

Table 3. Values of N_γ for strip smooth foundations

ϕ, deg.	ZEL	Hjiaj et al. (2005)	Bolton and Lau (1993)	Michalowski (1997)	Kumar and Kouzer (2007)
10	0.3	0.3	0.29	0.43	0.31
20	1.8	1.62	1.6	2.34	1.74
30	8	7.85	7.74	11.03	8.47
40	43	44.1	44	61.09	50.38
45	118	119.44	120	165.67	141.93

Table 4. Values of N_γ for circular rough foundations

ϕ, deg.	Present work based on the Triangular Rigid Cone (Bolton and Lau, 1993)	Present work based on the Lundgren and Mortensen assumption (1953)	Bolton and Lau (1993)	Kumar and Ghosh (2003)
10	1.3	0.5	1.37	0.27
20	6.25	2.5	6.04	1.96
30	34	14	31.9	12.79
40	250	120	238	111.05
45	830	400	803	420

Table 5. Values of N_γ for circular smooth foundations

ϕ, deg.	Present work	Bolton and Lau (1993)	Kumar and Kouzer (2003)
10	0.25	0.21	0.21
20	1.4	1.3	1.28
30	7.5	7.1	7.13
40	54	51	50.27
45	170	160	170

The values of the bearing capacity factor, N_γ, were also calculated by the ZEL method. For rough base foundations, the assumption of Bolton and Lau (1993) was considered for the analyses. Tables 2 and 3 show the bearing capacity factor computed by the ZEL method in comparison with other methods for strip foundations with smooth and rough bases, whereas Tables 4 and 5 present similar results for smooth and rough circular foundations. It is worth mentioning that many researchers have computed the bearing capacity factor N_γ for certain cases, e.g., for rough base strip foundations. There are very few

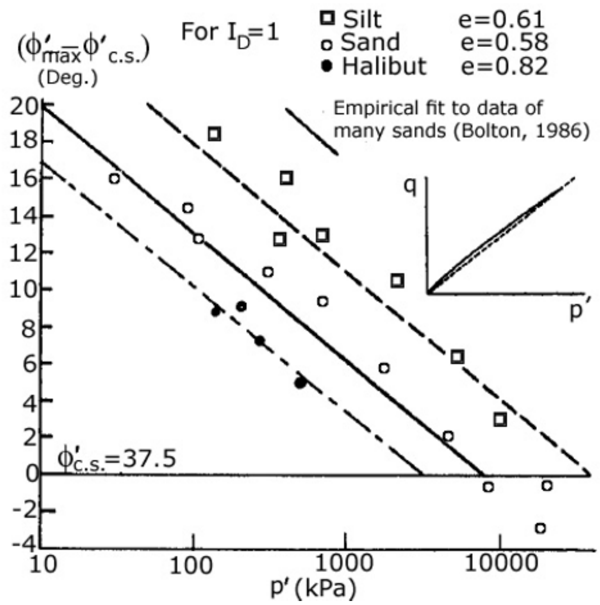

Fig. 4. Soil testing data from Bolton and Lau (1989) showing stress level dependence of friction angle

cases known to the authors in which all problem types (i.e., axisymmetric and plane strain problems for smooth and rough base foundations) are investigated, as the case of Bolton and Lau (1993) [5]. Comparisons have been made with different authors for different problems. It can be seen that the computed values are within the typical range suggested by different authors.

Bearing Capacity with Stress Level Consideration

As was stated earlier, the actual bearing capacity of foundations can be obtained by considering the stress level dependent nature of the subsoil medium. To investigate this effect and show the ability of the ZEL method to determine the actual bearing capacity of shallow foundations, an experimental study on different size foundations is presented and modeled with the aid of the ZEL method.

Bolton and Lau (1989) performed a centrifuge tests over two different soil types at different accelerations to model large-scale foundations in the laboratory [9]. Two types of dense soils, i.e., dense silt and dense sand were used in their program with a density of 16.5 kN/m^3 and 16.7 kN/m^3, respectively, and $\phi_{c.s.}$ equal to 37.5° for both soil types. By utilizing the variations of soil friction angle with confining pressure, shown in Fig. 4, the actual bearing capacity of different size foundations has been computed using the ZEL method with stress level dependence consideration. The assumption of Lundgren and Mortensen (1953) was adapted for the analyses for a rough base circular foundation [31].

Figure 5 shows the ZEL net for a circular foundation of 5.0 m diameter over the tested silt along with the variation of soil friction angle as a function of stress level. It is evident that maximum friction angle is achieved in regions experiencing lower pressures, i.e., near the stress free surface of the soil at the right side. Variations of soil friction angle ranged between the critical state friction angle and the peak friction angle obtained from the test results.

Table 6 shows the computed values of the bearing capacity factor N_γ, in comparison with experimental results. In the same table, the equivalent values of soil friction angle to provide similar values of N_γ are shown. It is obvious that equivalent soil friction angles increase with decrease in foundation size.

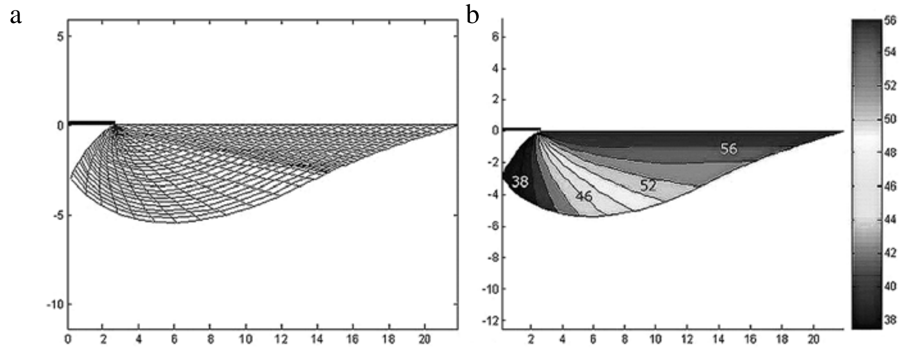

Fig. 5. Circular rough-base foundation of 5.0 m diameter on dense silt: a) ZEL net and b) variation of soil friction angle

Table 6. Computed and experimental results of the bearing capacity factor N_γ of rough-base circular foundations on dense silt and sand

D, m	Soil Type	N_γ				$\phi_{eq.}$, deg.
		experiment	ZEL	for ϕ_{peak} (ZEL)	for $\phi_{c.s.}$ (ZEL)	
5.0	Sand	237.6	195	>2000*	146	40.0
5.0	Silt	378.7	405	>2000	146	42.0
3.0	Silt	480.5	515	>2000	146	43.0
1.42	Silt	920	860	>2000	146	45.5

* Computations were very hard due to very high friction angles

As a result, the ZEL method can be reasonably applied for actual prediction of the bearing capacity of shallow foundations considering stress level dependence of soil friction angle. Another important conclusion is that using the conventional methods in the determination of the bearing capacity of shallow foundations is over-conservative if a critical state friction angle is assumed to be constantly distributed in the soil mass, whereas it provides unsafe and relatively high values if a uniformly distributed peak friction angle is assumed.

There are also some design charts developed by the authors of this paper to relate the actual values of the bearing capacity factor, N_γ, to foundation size by incorporating the relationship between soil friction angle and stress level suggested by Bolton (1986) [5]. This equation is as follow [5]:

$$\phi_{max} = \phi_{c.s.} + 5I_R \qquad \text{(in plane-strain condition)} \tag{8a}$$

$$\phi_{max} = \phi_{c.s.} + 3I_R \qquad \text{(in triaxial condition)} \tag{8b}$$

$$I_R = D_r(Q - \ln(\sigma)) - R. \tag{8c}$$

In these equations ϕ_{max} is the maximum mobilized friction angle, $\phi_{c.s.}$ is the critical state friction angle, I_R is dilatancy index, D_r is soil relative density (in decimals), σ is the effective stress (in kPa), and Q and R are constants. Bolton (1986) recommended to use Q = 10 and R = 1. Based on Bolton's (1986) equations, the developed charts are presented in Figs. 6 and 7, which can be used when soil relative density and critical state friction angle are determined from standard laboratory shear tests. The design charts show that while the effect of foundation size is significant for relatively small foundations, this effect becomes less significant with increasing foundation size beyond relatively large sizes.

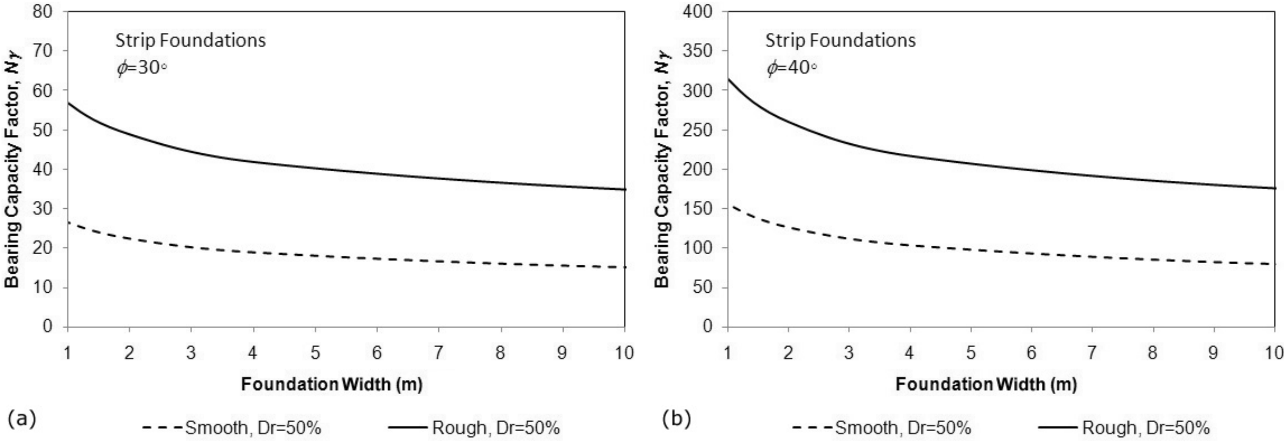

Fig. 6. Variation of the bearing capacity factor N_γ for strip foundations with smooth and rough bases: a) $\phi_{c.s.}$ = 30° and D_r = 50% and b) $\phi_{c.s.}$ = 40° and D_r = 50%

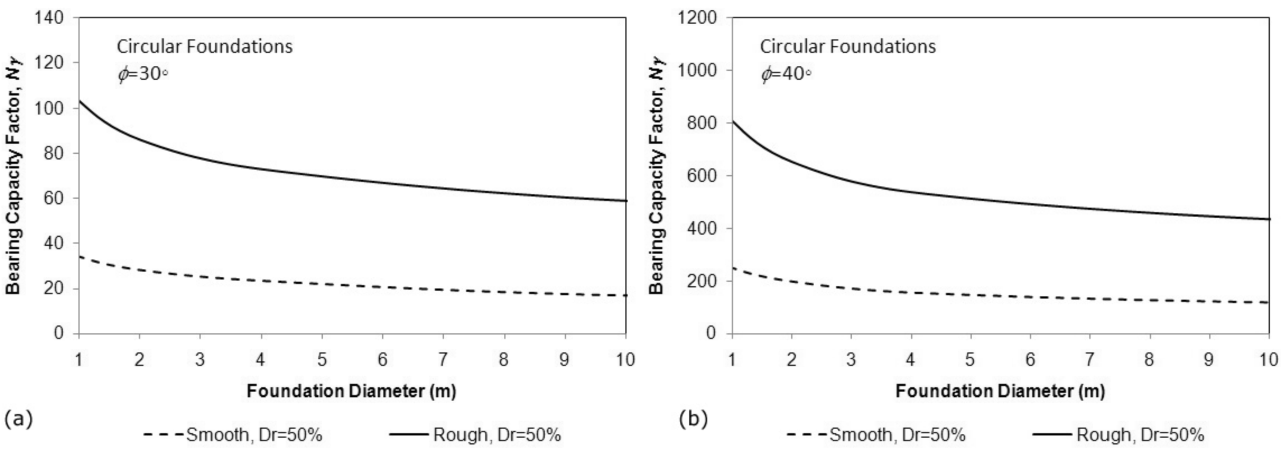

Fig. 7. Variation of the bearing capacity factor N_γ for circular foundations with smooth and rough bases: a) $\phi_{c.s.}$ = 30° and D_r = 50% and b) $\phi_{c.s.}$ = 40° and D_r = 50%

Conclusions and Discussion

In this paper the dependence of soil shear strength on the stress level was presented first and its influence on the bearing capacity of shallow foundations was explained. Next, the method of zero extension lines was introduced and the ZEL equations were utilized for the purpose of this study in which the effect of stress level on soil friction angle was considered. Later, the bearing capacity factors for shallow foundations were computed with and without considering the stress level effect on soil friction angle. The stress level dependence and its effect were studied by comparison with an existing experimental program on different size foundations. The computed results of the bearing capacity factor N_γ showed that the ZEL method is well capable of capturing the stress level dependency effect and provide more realistic results. The results also showed that the ZEL method with stress level consideration can provide actual estimation of the bearing capacity factor N_γ for shallow foundations. It is expected from this approach to prevent over-conservative or unsafe values resulting from the assumption of a uniformly distributed friction angle in the entire soil body under study. For practical purposes some design charts were provided in which the effect of foundation size on the bearing capacity factor N_γ was considered based on Bolton's (1986) equations for stress-level-dependent variations of soil friction angle.

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